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Long Run Risk Model and Equity Premium Puzzle in Thailand

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Abstract

This paper shows that the long-run risk model of Bansal and Yaron (2004) can potentially solve the equity premium and risk-free rate puzzles in Thailand. In particular, the calibrated values of the risk aversion and the elasticity of intertemporal substitution are empirically plausible. Risk decomposition results indicate that both short-run and long-run risks are equally important risk components relevant to asset prices in Thai financial markets while news regarding economic uncertainty, represented by volatility risk, have only an inconsequential impact.

Keywords: equity premium puzzle; long-run risk model; long-run component risk; asset pricing; generalized method of moments

1. Introduction

Equity premium puzzle has been an important problem in financial economics since the seminal work of Mehra and Prescott (1985), who found that the observed equity premium in the United States is too large to be explained by the consumption-based asset pricing model of Lucas (1978) and Breeden (1979) with a plausible value of risk aversion coefficient. This failure to explain the equity premium implies that standard macroeconomic models are not rich enough to capture relevant financial risks faced by the investors and their corresponding prices of risks. There has been a large body of literature trying to solve the equity premium puzzle using a variety of approaches, e.g., Epstein and Zin (1989); Weil (1989); Bansal and Yaron (2004), most of which, except Bansal and Yaron (2004), were not so successful. One of the most promising papers is Bansal and Yaron (2004), which showed that the long-run risk model can explain the equity premium and risk-free returns in the United States reasonably well.

So far, most of the literature in Thailand has shown that the equity premium puzzle exists. Both Duangthong (2014) and Harnphattananusorn (2014) found the equity premium puzzle in Thailand while Sedthapinun (2000), who used earlier data, found no puzzle. Recently, Duangchaiyoosook and Ousawat (2021) also revisited the issue using more recent data and found that the equity premium still exists in Thailand. The main question of this paper is whether the long-run risk model of Bansal and Yaron (2004) can resolve the equity premium puzzle in Thailand

This paper calibrates the long-run risk model of Bansal and Yaron (2004) to solve the equity premium puzzle in Thailand. The key contribution of this paper is the empirical part not the theoretical model since the latter is exactly the same as in Bansal and Yaron (2004). This paper estimates the long-run processes with time-varying economic uncertainty using the quarterly data of consumption and dividend growth rates from Thailand. The estimated parameters are then used for model calibration, where time discount factor, relative risk aversion coefficient and elasticity of intertemporal substitution are chosen in order to match the unconditional expectation of logs of equity premium and risk-free rate with their empirical counterparts. The calibration result suggests that the long-run risk model of Bansal and Yaron (2004) can explain the equity premium and risk-free rate in Thailand reasonably well. In particular, the calibrated values of the model parameters, time discount factor, relative risk aversion coefficient and elasticity of intertemporal substitution, are in a plausible range. In addition, risk decomposition results indicate that both sort-run and long-run risks are equally important risk components relevant to asset prices. For example, the former component accounts for about 57% of the equity premium for the benchmark case with a possible value of the discount factor while the latter for about 42%. On the other hand, news regarding economic uncertainty, represented by volatility risk, have only an inconsequential impact.

Intuitively, the long-run risk model can explain the equity premium because it captures two additional risks, long-run risk, and time-varying economic uncertainty, through the exogenous long-run processes of consumption and dividend growth while the standard model captures only short-run or consumption risk. More importantly, the model can price those risks while the standard model cannot because it assumes that investors have recursive preferences (Kreps & Porteus, 1978; Epstein & Zin, 1989; Weil, 1989). The key feature of recursive preferences is the ability to disentangle the relative risk aversion coefficient and the elasticity of intertemporal substitution, which is a key restriction of the time-separable utility function, employed in Mehra and Prescott (1985). In other words, both the additional risks and recursive preferences are complementary to each other. Recursive preferences alone is not sufficient to solve the puzzle, as shown in Weil (1989) for the US case, and Duangthong (2014) and Duangchaiyoosook and Ousawat (2021) for the Thai case while the two additional risks would have had no role on the equity premium if the preferences were time-separable (with the elasticity of intertemporal substitution is the reciprocal of the risk aversion).

The remainder of the paper is organized as follows. Section 2 describes the long-run risk model and its asset pricing implications. The empirical estimation of the long-run processes are presented in section 3. Section 4 presents the calibration results and section 5 concludes the paper.

2. The Long-Run Risk Model of Bansal and Yaron (2004)

This section presents the long-run risk model of Bansal and Yaron (2004), whose two key ingredients are (i) the consumption-based asset pricing model with recursive preferences (Epstein & Zin, 1989; Weil, 1989), and (ii) long-run processes with time-varying economic uncertainty of consumption growth and dividend growth.

According to the consumption-based asset pricing model with Epstein and Zin recursive preferences, the representative consumer solves the following utility maximization problem:

$$V_t = \max_{C_t} U(C_t, E_t[V_{t+1}]) \tag{1}$$

subject to the budget constraint[1]

$$W_{t+1} = (W_t - C_t) R_{c,t+1}, \tag{2}$$

where C_t is consumption in period t, W_t is wealth at the beginning of period t, $U(C,V)$ is an aggregator function capturing the recursive nature of the preferences, and V_t is the value function representing the maximum utility attainable by the agent.

One key assumption here is the complete markets assumption in that the consumer has access to the complete set of assets, whose dividend is the aggregate consumption C_{t+1} and total return is $R_{c,t+1} = \dfrac{P_{c,t+1} + C_{t+1}}{P_{c,t}}$, where $P_{c,t}$ is the price of the portfolio in period t. Some of the assets may not be traded in the financial markets, however. This implies that the return on aggregate

[1] The budget constraint (2) matters here because the value function *V* is a function of *W*.

consumption $R_{c,t+1}$ does not need to be equal to the financial market return $R_{m,t+1}$, and $P_{c,t}$ is unobserved by econometricians.

Following Weil (1989), we assume that the aggregator function is given by

$$U(C,V)=\frac{\left((1-\beta)C^{1-\frac{1}{\psi}}+\beta(1+(1-\beta)(1-\gamma)V)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}}-1}{(1-\beta)(1-\gamma)} \tag{3}$$

where β is the time discount factor with $0<\beta<1$, γ is the relative risk aversion coefficient, and ψ is the elasticity of intertemporal substitution. The Euler equation for the utility maximization problem is

$$E_t\left[\left(\beta^{\frac{\psi(1-\gamma)}{\psi-1}}G_{c,t+1}^{\frac{\gamma-1}{\psi-1}}R_{c,t+1}^{\frac{1-\psi\gamma}{\psi-1}}\right)R_{c,t+1}\right]=1, \tag{4}$$

which implies that the log of the stochastic discount factor, $m_{t+1}=\log M_{t+1}$ with $M_{t+1}=\beta^{\frac{\psi(1-\gamma)}{\psi-1}}G_{c,t+1}^{\frac{\gamma-1}{\psi-1}}R_{c,t+1}^{\frac{1-\psi\gamma}{\psi-1}}$, is

$$m_{t+1}=\frac{\psi(1-\gamma)}{\psi-1}\log\beta+\frac{\gamma-1}{\psi-1}g_{c,t+1}+\frac{1-\psi\gamma}{\psi-1}r_{c,t+1}, \tag{5}$$

where $g_{c,t+1}\equiv\log G_{c,t+1}$ is the log of consumption growth, and $r_{c,t+1}=\log R_{c,t+1}$ is the log of return on the complete-markets portfolio. Importantly, the stochastic discount factor can price any asset whose returns are $R_{j,t+1}$ according to the standard pricing equation $E_t[M_{t+1}R_{j,t+1}]=1$. Note that lowercase letters in this paper refer to the natural logs.

The second key ingredient of the model is the exogenous long-run processes of the log of consumption growth $g_{c,t+1}$, the log of dividend growth $g_{d,t+1}$, unobserved persistent variable x_{t+1}, and time-varying economic uncertainty $\sigma^2_{c,t+1}$

$$g_{c,t+1} = \mu_c + x_t + \sigma_{c,t}\eta_{c,t+1}, \tag{6}$$

$$g_{d,t+1} = \mu_d + \phi x_t + \pi_d \sigma_{c,t}\eta_{c,t+1} + \varphi_d \sigma_{c,t}\eta_{d,t+1}, \tag{7}$$

$$x_{t+1} = \rho x_t + \varphi_x \sigma_{c,t}\eta_{x,t+1}, \tag{8}$$

$$\sigma^2_{c,t+1} = \sigma^2_c + \nu(\sigma^2_{c,t} - \sigma^2_c) + \varphi_\sigma \eta_{\sigma,t+1}, \tag{9}$$

where μ_c is the expected value of the log of consumption growth, μ_d is the expected value of the log of dividend growth, ϕ is the leverage ratio of the persistent variable x_t, π_d is the dividend-consumption exposure, φ_d is the volatility multiplier ratio of the log of dividend growth, ρ is the persistence of the growth process, φ_x is the volatility multiplier of the persistent variable, σ^2_c is the expected value of time-varying economic uncertainty, ν is the persistence of time-varying economic uncertainty; φ_σ is the volatility multiplier of the time-varying economic uncertainty, and all error terms, $\eta_{c,t+1}$, $\eta_{d,t+1}$, $\eta_{x,t+1}$, $\eta_{\sigma,t+1}$, are i.i.d. standard normal $N(0, 1)$ and uncorrelated with each other. To sum up, these processes have 10 parameters, μ_c, μ_d, ϕ, π_d, φ_d, ρ, φ_x, σ^2_c, ν, φ_σ, which are estimated in section 3.

The next step is to apply the standard first-order Taylor approximation, as in Campbell and Shiller (1988), to $r_{c,t+1} = \log\left(\frac{P_{c,t+1}+C_{t+1}}{P_{c,t}}\right)$:

$$r_{c,t+1} \approx k_{c,0} + k_{c,1} z_{c,t+1} - z_{c,t} + g_{c,t+1}, \tag{10}$$

where $z_{c,t} = \log\left(\frac{P_{c,t}}{C_t}\right)$ is the log of price-consumption ratio,

$k_{c,0} = \log\left(1+e^{\overline{z}_c}\right) - k_{c,1}\overline{z}_c$, $k_{c,1} = \frac{e^{\overline{z}_c}}{1+e^{\overline{z}_c}}$ and $\overline{z}_c$ is the mean of z_c.[2]

Following Bansal and Yaron (2004), we guess that the log of price-consumption ratio $z_{c,t}$ is a linear function of the unobserved persistent variable x_t, and time-varying economic uncertainty $\sigma^2_{c,t}$:

$$z_{c,t} = A_{c,0} + A_{c,1}x_t + A_{c,2}\sigma^2_{c,t}, \tag{11}$$

which can be verified by substituting (10) and (11) into the Euler equation (4), and then solving for $A_{c,0}$, $A_{c,1}$ and $A_{c,2}$. See the detailed derivation[3] in Appendix A.1.

A similar approach can be employed to derive the approximation of the log of financial-market return, $r_{m,t+1} = \log\left(\frac{P_{m,t+1}+D_{t+1}}{P_{m,t}}\right)$, where $P_{m,t+1}$ is the price of the financial market portfolio, and D_{t+1} is its dividend. The approximation of $r_{m,t+1}$ is

$$r_{m,t+1} \approx k_{m,0} + k_{m,1}z_{m,t+1} - z_{m,t} + g_{d,t+1}, \tag{12}$$

[2] The key part is $\log\left(1+Z_{c,t+1}\right) = \log\left(1+e^{z_{c,t+1}}\right) \approx \log\left(1+e^{\overline{z}_c}\right) - \left(\frac{e^{\overline{z}_c}}{1+e^{\overline{z}_c}}\right)\overline{z}_c + \left(\frac{e^{\overline{z}_c}}{1+e^{\overline{z}_c}}\right)z_{c,t+1}$, which employs the standard first-order Taylor approximation around any $\overline{z}_c$ that is not too far from $z_{c,t+1}$. If the price-consumption ratio were observed, it would have been easier to estimate $\overline{z}_c$. Following Bansal and Yaron (2004), we can find $\overline{z}_c$ by first setting its theoretical counterpart equal to the mean of $z_{c,t}$, i.e., $\overline{z}_c = E[z_{c,t}]$. Using this condition, we can then solve for $\overline{z}_c$ using the expectation of (11): $E[z_{c,t}] = \overline{z}_c = A_{c,0} + A_{c,2}\sigma^2_c$, where both $A_{c,0}$ and $A_{c,2}$ are also functions of $\overline{z}_c$.

[3] An implicit assumption needed for the derivation is log-normality of consumption growth and the return on the complete-markets portfolio.

where $z_{c,t} = \log\left(\frac{P_{m,t}}{D_t}\right)$ is the log of price-dividend ratio, $k_{m,0} = \log\left(1+e^{\bar{z}_m}\right) - k_{m,1}\bar{z}_m$, $k_{m,1} = \frac{e^{\bar{z}_m}}{1+e^{\bar{z}_m}}$, and $\bar{z}_m$ is the average of the log of price-dividend ratio, which is observed.[4] Again, the log of price-dividend ratio, $z_{m,t}$, is guessed to be a linear function of the unobserved persistent variable x_t, and time-varying economic uncertainty $\sigma_{c,t}^2$:

$$z_{m,t} = A_{m,0} + A_{m,1}x_t + A_{m,2}\sigma_{c,t}^2, \tag{13}$$

which can be verified by substituting (12) and (13) into the Euler equation (4) and then solving for $A_{m,0}$, $A_{m,1}$ and $A_{m,2}$. See the detailed derivation[5] in Appendix A.2.

As in the standard asset pricing literature, to calibrate the model, we need to derive key asset pricing equations as unconditional expectations, one for equity premium $r_{m,t+1} - r_{f,t+1}$, and one for risk-free rate $r_{f,t+1}$. The unconditional expectation of the equity premium can be written in terms of model parameters and observed statistics as the following:

$$E\left[r_{m,t+1} - r_{f,t+1}\right] = \lambda_c\beta_c + \lambda_x\beta_x + \lambda_\sigma\beta_\sigma - \frac{1}{2}E\left[Var_t\left[r_{m,t+1}\right]\right], \tag{14}$$

where

$$E\left[Var_t\left[r_{m,t+1}\right]\right] = (k_{m,1}A_{m,1}\varphi_x\sigma_c)^2 + (\pi_d\sigma_c)^2 + (\varphi_d\sigma_c)^2 + (k_{m,1}A_{m,2}\varphi_\sigma)^2, \tag{15}$$

[4] Since $z_{m,t}$ is observed, $\bar{z}_m$ can be directly estimated using the average of the log of price-dividend ratio. This is clearly different from the price-consumption ratio, which is unobserved.

[5] An implicit assumption needed for the derivation is log-normality of consumption growth, complete-market return, and financial-market return.

Note that the equity premium can be decomposed into risk components, β, and corresponding prices of risks, λ. In particular, $\beta_c = \pi_d \sigma_c^2$ and $\lambda_c = \gamma$ denote the short-run risk (or consumption risk) and its price, $\beta_x = \varphi_x \sigma_c^2 k_{m,1} A_{m,1}$ and $\lambda_x = \dfrac{\varphi_x(\psi\gamma - 1)k_{c,1}}{\psi(1-\rho k_{c,1})}$ denote the long-run risk and its price, and $\beta_\sigma = \varphi_\sigma^2 k_{m,1} A_{m,2}$ and $\lambda_\sigma = \dfrac{\psi\gamma - 1}{\psi - 1} k_{c,1} A_{c,2}$ denote the volatility risk and its price. The detailed derivations are in Appendix A.3. Note also that prices of the long-run risk, λ_x, and the volatility risk, λ_σ, would have been zero if $\psi\gamma = 1$. In other words, both types of risks would have had no role on the equity premium if the preferences were time-separable (with the elasticity of intertemporal substitution is the reciprocal of the risk aversion). That is, recursive preferences are necessary for pricing the long-run risk and the volatility risk.

Similarly, the unconditional expectation of the log of risk-free return can be written in terms of model parameters and observed statistics as the following:

$$E\left[r_{f,t+1} \right] = -\log\beta + \frac{\mu_c}{\psi} + \frac{\psi\gamma - 1}{\psi(1-\gamma)} E\left[r_{c,t+1} - r_{f,t+1} \right] + \frac{\psi\gamma - 1}{2\psi(1-\gamma)} E\left[Var_t\left[m_{t+1} \right] \right], \quad (16)$$

where

$$E\left[r_{c,t+1} - r_{f,t+1} \right] = \lambda_c \sigma_c^2 + \lambda_x k_{c,1} A_{c,1} \varphi_x \sigma_c^2 + \lambda_\sigma k_{c,1} A_{c,2} \varphi_\sigma^2 - \frac{1}{2} E\left[Var_t\left[r_{c,t+1} \right] \right], \quad (17)$$

$$E\left[Var_t\left[r_{c,t+1} \right] \right] = (1 + (k_{c,1} A_{c,1} \varphi_x)^2)\sigma_c^2 + (k_{c,1} A_{c,2})^2 \varphi_\sigma^2, \quad (18)$$

$$E\left[Var_t\left[m_{t+1} \right] \right] = \lambda_c^2 \sigma_c^2 + \lambda_x^2 \sigma_c^2 + \lambda_\sigma^2 \varphi_\sigma^2. \quad (19)$$

The detailed derivations are in Appendix A.4.

The main purpose of this paper is to calibrate the long-run risk model, using the asset pricing equations (14) and (16), which depend on 13 parameters, $\beta, \gamma, \psi, \mu_c, \mu_d, \phi, \pi_d, \varphi_d, \rho, \varphi_x, \sigma_c^2, \nu, \varphi_\sigma$. Note that the first three parameters are calibrating parameters, which will not be estimated directly, while the last 10 of them will be estimated using consumption and dividend data as in the next section.

3. Estimation of the Long-Run Processes

This section estimates the long-run processes (6)-(9), which contain $K = 10$ structural parameters. To be able to identify and estimate these parameters using the generalized method of moments[6] or GMM (Hansen, 1982), we derive $I = 12$ moment conditions of observed variables, logs of consumption and dividend growth, including their own and cross moments up to the fourth order both with contemporaneous and lagged variables. See the exact forms of these moments in Appendix A.5.

More formally, let $\Theta \equiv \{\mu_c, \mu_d, \phi, \pi_d, \varphi_d, \rho, \varphi_x, \sigma_c^2, \nu, \varphi_\sigma\}$ be the set of structural parameters to estimate. Let $E[f_i(\boldsymbol{\upsilon}, \Theta)] = 0$ be the i^{th} moment condition for $i = 1, ..., I$, where $\boldsymbol{\upsilon} = (g_c, g_d)$ are random variables representing consumption and dividend growth rates. Staking all of them together, we can write the (theoretical) moment conditions in a vector form as the following.

$$E[\boldsymbol{f}(\boldsymbol{\upsilon}, \Theta)] = 0, \tag{20}$$

[6] One advantage of the GMM approach in this case is that it is a natural setting to deal with unobserved persistent variables, e.g., x_t, by treating it as a random variable with finite moments.

where $\boldsymbol{f}(\boldsymbol{\upsilon},\Theta)$ is the vector of I moment conditions. The GMM approach is to find a set of parameters, $\widehat{\Theta}$, that solves the following minimization problem:

$$\widehat{\Theta} = \arg\max_{\Theta} \overline{\boldsymbol{f}}(\hat{\boldsymbol{\upsilon}},\Theta)'\widehat{\boldsymbol{A}}\overline{\boldsymbol{f}}(\hat{\boldsymbol{\upsilon}},\Theta), \tag{21}$$

where $'$ denotes the matrix transpose operator, and $\widehat{\boldsymbol{A}}$ is the weighting matrix. In addition, $\overline{\boldsymbol{f}}(\hat{\boldsymbol{\upsilon}},\Theta) \equiv \left[\ \overline{f}_1(\hat{\boldsymbol{\upsilon}},\Theta),...,\overline{f}_I(\hat{\boldsymbol{\upsilon}},\Theta)\right]$ is the vector of the averages of the moment conditions with

$$\overline{f}_i(\hat{\boldsymbol{\upsilon}},\Theta) = \frac{\sum_{t=1}^{T} f_i(\hat{\boldsymbol{\upsilon}}_t,\Theta)d_i(t)}{\sum_{t=1}^{T} d_i(t)}, \tag{22}$$

where $d_i(t)=1$ if period-t data, $\hat{\boldsymbol{\upsilon}}_t$ for $t=1,...,T$, is applicable to the i th moment condition, and equals to zero otherwise.[7] Our weighting matrix is based on the Newey-West estimator (Newey and West, 1987) with $L=2$ lags, which is always positive semi-definite, as follows:

$$\widehat{\boldsymbol{A}} = \left[\hat{\boldsymbol{\Gamma}}(0) + \sum_{l=1}^{L}\left(1-\frac{l}{1+L}\right)\left(\hat{\boldsymbol{\Gamma}}(l)+\hat{\boldsymbol{\Gamma}}(l)\right)\right]^{-1}, \tag{23}$$

where $\hat{\boldsymbol{\Gamma}}(l) \equiv \left[\ \overline{\Gamma}_{ij}(l)\right]$ is an $I \times I$ matrix with

$$\overline{\Gamma}_{ij}(l) = \frac{\sum_{t=1}^{T} f_i(\hat{\boldsymbol{\upsilon}}_t,\Theta) f_i(\hat{\boldsymbol{\upsilon}}_{t-l},\Theta)d_i(t)d_j(t-l)}{\sum_{t=1}^{T} d_i(t)d_j(t-l)}, \text{ for } l=0,1,...,L \tag{24}$$

[7] These dummies are needed because moment conditions with different number of lags will have a different number of relevant observations.

where $d_j(t) = 0$ if $t \leq 0$. The asymptotic variance-covariance matrix of the estimated parameters can be calculated as the following.

$$\widehat{VAR}\left[\,\widehat{\Theta}\,\right] = \frac{\left(\hat{G}'\,\hat{A}\hat{G}\right)^{-1}}{T}, \tag{25}$$

where $\widehat{\boldsymbol{G}} = \left[\,\bar{G}_{ik}\,\right]$ is an $I \times K$ matrix with

$$\bar{G}_{ik} = \frac{\sum_{t=1}^{T} \frac{\partial f_i(\widehat{\boldsymbol{\upsilon}}_t, \Theta)}{\partial \theta_k} d_i(t)}{\sum_{t=1}^{T} d_i(t)}, \tag{26}$$

which is the empirical counterpart of the gradient matrix $\frac{\partial E\left[\, f_i(\hat{\upsilon}, \Theta)\right]}{\partial \theta_k}$. The standard error of each estimated parameter can be discovered from the corresponding diagonal element of the estimated variance-covariance matrix.

The baseline estimation of this paper uses the quarterly per capita consumption and dividend data from 2000 to 2019. This choice is to avoid the complication of the financial crisis of 1997. As a robustness check, we also perform the estimation and calibration with longer data from 1994 to 2019. Nominal aggregate consumption data are taken from the national income account, generated by the Office of the National Economic and Social Development Board (NESDB) while nominal dividend data are computed using cash and stock dividends from all stocks available in each period from the Stock Exchange of Thailand.[8] These nominal variables are adjusted to be real variables using the quarterly Consumer Price Index (CPI) of Thailand, generated by Bureau of Trade and Economic Indices. Per capita variables are adjusted using the population data of Thailand from the Department of

[8] There are two main reasons why we should use both cash and stock dividends in this paper: (i) the equity premium is calculated using the total return, based on capital gains, cash dividend and stock dividend. (ii) this model does not require the equality of dividend and consumption.

Provincial Administration.[9] Real quarterly (per capita) consumption growth rate, $G_{c,t}$, and real (per capita) dividend growth rate, $G_{d,t}$, are calculated using year-on-year measurement of the real quarterly data, which makes the units of all growth rates as per annum. The year-on-year calculation is to adjust for potential seasonal effects. The average of log of real (per capita) consumption growth rate, $\overline{g}_c$, over the period of 2000-2019 is approximately 0.038 per annum (with standard deviation of 0.019) while it is about 0.139 per annum (with standard deviation of 0.433) for the real (per capita) dividend growth rate, $\overline{g}_d$. See table 1.

This paper employs a two-step GMM estimation method, where the weighting matrix of the first step is the identity matrix while the second step is calculated using (23) based on the first-step estimated parameters. In principle, all ten parameters should be estimated freely but that unconstrained estimation led to a non-stationary result, i.e., $|\hat{\nu}| > 1$.[10] It is, therefore, reasonable to set the persistent parameter of economic uncertainty not only to be less than one but sufficiently close to one as well. In particular, we set $\hat{\nu} = 0.999$.

Table 1: Summary statistics of log of consumption growth, log of dividend growth, log of price-dividend ratio, log of financial market return, and log of risk-free return.

Parameters	2000-2019		1994-2019	
	mean	standard deviation	mean	standard deviation
log of consumption growth ($g_{c,t+1}$)	0.038	0.019	0.034	0.028

[9] The original population data are available in annual frequency only. We use a simple interpolation method to calculate the quarterly version.

[10] We thank an anonymous referee who reminds us about the non-stationary result in an earlier draft of the paper.

log of dividend growth ($g_{d,t+1}$)	0.139	0.439	0.056	0.563
log of price-dividend ratio ($z_{m,t+1}$)	4.917	0.748	5.103	0.833
log of financial market return ($r_{m,t+1}$)	0.050	0.248	-0.017	0.323
log of risk-free return ($r_{f,t+1}$)	-0.002	0.019	0.007	0.024

Note: All variables are calculated using year-on-year measurement of quarterly data

The baseline estimation results are reported in the second column of table 2. All parameter estimates are statistically significant with $p < 0.05$ except $\widehat{\varphi}_{\sigma}$. The significance of the persistence of the growth process, $\widehat{\rho} = 0.341$, and the volatility multiplier of the persistent variable, $\widehat{\varphi}_x = 0.386$, confirm that consumption growth process in Thailand is a long-run process not an i.i.d. With the significance of the leverage ratio of the persistent variable, $\widehat{\phi} = 1.672$, the dividend growth process is also a long-run process. In addition, the magnitude of the estimate, $\widehat{\phi} > 1$, is in line with the result of Abel (1999), who argued that the leverage ratio from the financial markets should be larger than one and consumption and dividend should be treated as two distinct processes, which is different from the consumption-based asset pricing model of Lucas (1978). Insignificance of the volatility multiplier of the time-varying economic uncertainty, $\widehat{\varphi}_{\sigma} = 0.006$, indicates that time-varying economic uncertainty does not affect consumption growth process significantly in Thailand. This insignificant result is similar to the one in Bansal et al. (2016).

We next compare our baseline estimates with the US case from Bansal et al. (2016), who estimated the long-run risk model using GMM as well. Most of the estimated parameters are comparable, except $\widehat{\sigma}_c^2$, $\widehat{\mu}_d$ and

$\widehat{\varphi}_d$, as shown in table 2. The differences for dividend-related parameters, $\widehat{\mu}_d$ and $\widehat{\varphi}_d$, should be expected since real per capita dividend growths from both countries could potentially be distinct. The fact that $\widehat{\sigma}_c^2$ is significantly larger for the Thai economy than the US indicates that the US may be able to manage consumption risks better than the Thai economy. Unfortunately, testing this hypothesis is beyond the scope of this paper. Note also that the persistence of the growth process for Thailand, $\widehat{\rho}$, is appropriately smaller than the US one. We conjecture that this difference results from the large difference in the variance of consumption growth, $\widehat{\sigma}_c^2$, since consumption growth process depends on it.

The baseline estimates are fairly different from the ones using the extended data from 1994 to 2019. See the second and the third columns of table 2. It is evident that all dividend-related parameters, $\widehat{\mu}_d$, $\widehat{\phi}$, $\widehat{\varphi}_d$ and $\widehat{\pi}_d$, are distinct. This may result from the fact that Thai firms have changed their dividend policies after the financial crisis of 1997 (Ronapat and Evans, 2005). Another different parameter is the variance of consumption growth, $\widehat{\sigma}_c^2$. Unfortunately, we do not have a reasonable explanation for this difference. Note that J statistics (reported at the bottom of the table) imply that the overidentifying restrictions are rejected for all three cases. As in Bansal et al. (2016), we still use the estimated parameters to calibrate the long run risk model even though the overidentifying restrictions are rejected.

Table 2: Estimated parameters of the long-run processes with time-varying economic uncertainty.

Estimated Parameters	Thailand case		US case
	2000-2019	1994-2019	BYK(2016)
$\widehat{\rho}$	0.341***	0.376***	0.874
	(0.013)	(0.070)	

$\hat{\varphi}_x$	0.386***	0.523***	0.166
	(0.008)	(0.066)	
$\hat{\sigma}_c^2$	0.124***	0.014***	0.001
	(0.024)	(0.003)	
$\hat{\mu}_c$	0.038***	0.034***	0.014
	(0.003)	(0.003)	
$\hat{\mu}_d$	0.139**	0.056	0.011
	(0.053)	(0.067)	
$\hat{\phi}$	1.672***	3.945*	2.51
	(0.204)	(1.587)	
$\hat{\varphi}_d$	0.698*	3.794***	5.12
	(0.276)	(0.429)	
$\hat{\pi}_d$	0.857***	3.084***	0.6
	(0.034)	(0.374)	
$\hat{\nu}$	0.999	0.999	0.922
	(N.A.)	(N.A.)	
$\hat{\varphi}_\sigma$	0.006	0.001	0.00000349
	(0.241)	(1.298)	
J-stat	26.685	34.688	231.5
p-value	0.000	0.000	0.000

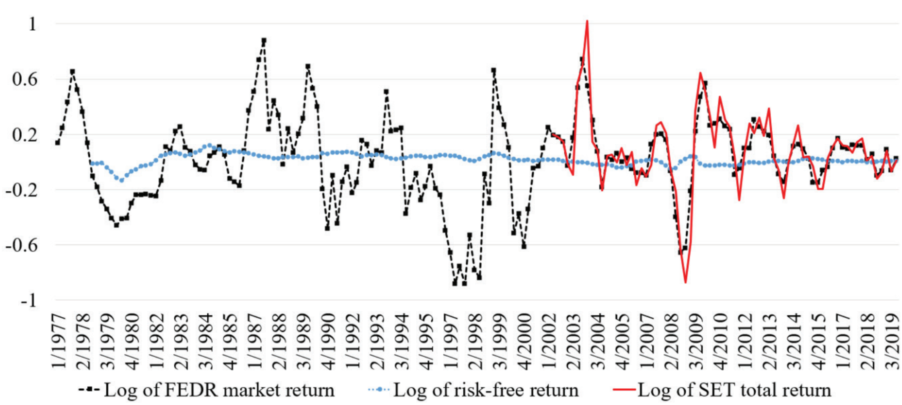
Note: Standard errors are in parenthesis and * $p<0.05$, ** $p<0.01$, *** $p<0.001$. The weighting matrix is the Newey-West with 2 lags. *J*-Stat and *p*-values in the last two rows are for the over-identification tests.

4. Calibration Results

This section calibrates the model by choosing time discount factor, β, relative risk aversion coefficient, γ, and elasticity of intertemporal substitution, ψ, to match the unconditional expectations of logs of equity premium and risk free rate, (14) and (16), based on relevant sample statistics and estimated parameters with their empirical counterparts.

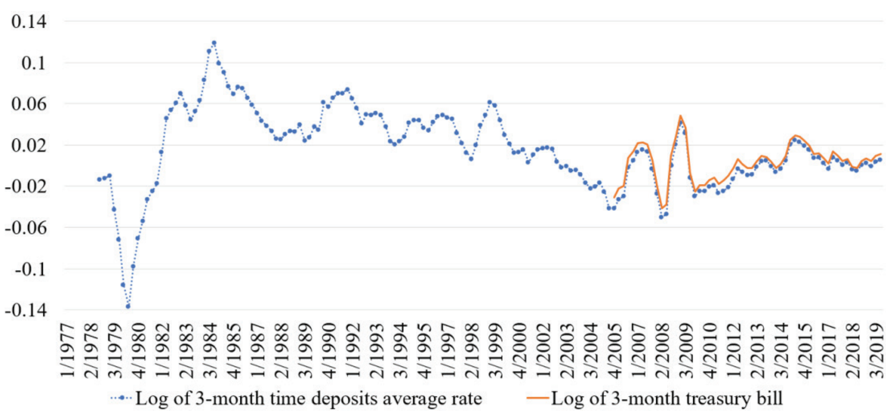
The observed financial market returns, $R_{m,t+1}$, are taken from the Financial and Economic Data for Research (FEDR) at the University of the Thai Chamber of Commerce (UTCC), which collects and adjusts financial data from the Stock Exchange of Thailand (SET). The FEDR returns are constructed using a similar framework to the CRSP market returns from the Center for Research in Security Prices at the University of Chicago. In particular, the total returns for each individual stock are calculated as the sum of the returns from the capital gains, cash dividends and stock dividends, taking into account stock split/reverse. The FEDR market returns, henceforth market returns, are the returns of the value-weighted portfolio of all stocks in the SET. The key advantage of the FEDR market returns over the SET total returns provided by the SET is that the former returns are available since the beginning of the SET (April, 1975) while the latter ones are available only after January, 2002. To be consistent with the calculation of consumption and dividend growth, the real market returns here are calculated using year-on-year measurement of the real quarterly returns. See Appendix A.6 for derivations. Figure 1 shows log of quarterly market returns, log of quarterly SET total returns, and log of quarterly risk-free returns (all units are per annum). The average of log of real market returns over the period 2000-2019 is approximately 0.050 per annum, as shown in table 1. This number is slightly smaller than the average annual real market returns of 0.067 per annum.

Figure 1: Log of real market returns (1977-2019), log of SET real total returns (2002-2019), and log of real 3-month time deposits average rate of main five Thai Commercial Banks (1978- 2019).



The observed risk-free returns, $R_{f,t+1}$, are the 3-month time deposits average returns of the five main Thai commercial banks, including Bangkok Bank, Krungthai Bank, Siam Commercial Bank, Kasikorn Bank and Bank of Ayudhya, taken from the Bank of Thailand (BOT). We need to use the deposit rate instead of the 3-month treasury bill returns as in the international literature (e.g., Mehra & Prescott, 1985) because the treasury data of Thailand are consistently available only after 2005 while the deposit rates are available since 1978. In addition, both rates are reasonably close when both are available, as evident in figure 2. The real risk-free returns are also calculated using year-on-year measurement of the real quarterly returns. The average of log of real risk-free returns over the period 2000-2019 is approximately -0.002 per annum, as shown in table 1. This number is slightly smaller than the average annual real risk-free returns of -0.001 per annum.

Figure 2: Log of real 3-month time deposits average rate of the five main Thai Commercial Banks (1978-2016) and log of 3-month treasury bill returns (2005-2019).



Note: All returns are calculated using year-on-year measurement of the corresponding real quarterly returns

Another key statistic is the average of the log of price-dividend ratio, which is the estimate of $\overline{z}_m$. We compute the price-dividend ratio, using the quarterly observed cash and stock dividends and market price. The average of the log of price-dividend ratio over the period 2000-2019 is approximately 4.919, which is slightly larger than the value of 3.404 for the US (Bansal et al., 2016).

We are now ready to calibrate the model. There are three parameters to calibrate with two equations. Therefore, we need to predetermine one parameter. Following the literature, we calibrate the model by solving equations (14) and (16) jointly for risk aversion coefficient γ and elasticity of intertemporal substitution ψ, for each particular value of the time discount factor $\beta = 0.97, 0.98, 0.99$.

Table 3 shows the calibration results, which suggest that the long-run risk model can resolve equity premium and risk-free rate puzzles in Thailand. For the baseline case, calibrated values of risk aversion coefficient γ and elasticity of intertemporal substitution ψ are empirically plausible, for each time discount rate. Risk aversion is between 1.02-1.10, which is in a plausible range suggested by empirical studies (Mehra & Prescott, 1985) while the elasticity of intertemporal substitution is between 0.34-0.37, which is also empirically reasonable (see, e.g., Hall, 1988; Campbell, 1999). See the second and third columns of table 3. Note that the average of log of consumption-price ratio corresponding to each set of calibrated parameters is between 2.38-2.57, which is much lower than the average of log of dividend-price ratio of 4.92. The calibration results using data from 1994-2019 also lead to a similar conclusion. That is, the model can be calibrated to match equity premium and risk-free returns with reasonable parameter values.

Table 3: Calibrated values of risk aversion γ and elasticity of intertemporal substitution ψ.

β	2000-2019			1994-2019		
	γ	ψ	$\bar{z}_c$	γ	ψ	$\bar{z}_c$
0.97	1.100	0.341	2.376	3.395	1.968	3.528

0.98	1.060	0.355	2.477	3.596	1.098	3.876
0.99	1.016	0.366	2.570	3.812	0.809	4.294

We can also decompose risk components of the model using (14). As discussed earlier in section 2, the equity premium can be decomposed into short-run risk component, $\lambda_c\beta_c$, long-run risk component, $\lambda_x\beta_x$, and volatility risk component, $\lambda_\sigma\beta_\sigma$, each of which can be calculated using estimated and calibrated parameters and observed statistics. The baseline results, presented in table 4, suggest that both short-run and long-run risk are key contributors to the equity premium. For example, for the case with β = 0.97, short-run risk accounts for about 57% for the equity premium while long-run risk explains roughly 42%. This is consistent with Bansal and Yaron (2004), who find that short-run and long-run risks are the important sources of the variance of pricing kernel. Again, the results are robust with regards to the extension of data to 1994-2019. See table 4.

Table 4 : Contribution of short-run, long-run and volatility risks.

β	2000-2019			1994-2019		
	short-run	long-run	volatility	short-run	long-run	volatility
0.97	0.117	0.085	0.002	0.147	0.094	-0.0232
	(57%)	(42%)	(1%)	(67%)	(43%)	(-10%)
0.98	0.113	0.075	0.001	0.158	0.078	-0.031
	(60%)	(39%)	(1%)	(77%)	(38%)	(-15%)
0.99	0.108	0.068	0.0003	0.168	0.068	-0.042
	(61%)	(38%)	(1%)	(87%)	(35%)	(-22%)

In addition, the pricing formulation in (14) enables us to decompose each risk component into the corresponding risk measure, β, and its market price, λ. The baseline results, presented in table 5, show that the absolute values of short-run and long-run risk measures and market prices of risks are comparable. This again suggests that both short-run and long-run risks are crucial for asset prices. However, the short-run components are positive, as expected, while the long-run ones are both negative. On the other hand, for the alternative data of 1994-2019, the long-run components are both positive. Mechanically, price of long-run risk will be negative when the product of risk aversion and intertemporal elasticity of substitution, $\gamma\psi$, is smaller than one, and vice versa.[11] Based on the calibration results in table 3, the product is smaller than one for the 2000-2019, which is mainly driven by the fact that the estimate for intertemporal elasticity of substitution is significantly less than one, and larger than one for the 1994-2019.

Table 5: Risk measures and their prices for 2000-2019.

β	Risk Measures			Prices of Risks		
	short-run	long-run	volatility	short-run	long-run	volatility
0.97	0.106	-0.091	0.002	1.100	-0.941	1.236
0.98	0.106	-0.082	0.002	1.060	-0.912	0.792
0.99	0.106	-0.076	0.001	1.016	-0.899	0.022

Table 6: Risk measures and their prices for 1994-2019.

β	Risk Measures			Prices of Risks		
	short-run	long-run	volatility	short-run	long-run	volatility
0.97	0.04	0.04	0.0001	3.395	2.311	-186.825
0.98	0.04	0.04	0.0001	3.596	2.177	-265.815
0.99	0.04	0.03	0.0001	3.812	2.112	-413.281

6. Conclusion

This paper calibrates the long-run risk model of Bansal and Yaron

[11] The same product also determines the sign of price of volatility risk, which will be positive when the product of risk aversion and intertemporal elasticity of substitution, $\gamma\psi$, is smaller than one, and vice versa. Since the contribution of volatility risk is inconsequential, we will not discuss it here.

(2004) to match both the equity premium and the risk-free returns in Thailand. To do so, we estimate the long-run processes of consumption and dividend growths. The estimation results indicate that consumption and dividend indeed follow long-run processes.

The calibration results confirm that the long-run risk model can potentially solve the equity premium and risk-free rate puzzles in Thailand. In particular, the calibrated values of the risk aversion and the elasticity of intertemporal substitution are empirically plausible. Moreover, risk decomposition results indicate that both short-run and long-run risks are equally important risk components relevant to Thai financial markets while news regarding economic uncertainty, represented by volatility risk, have only an inconsequential impact.

One limitation of this paper is that it focuses only on the equity premium and risk-free rate implications of the model even though the model is capable of explaining a number of financial phenomena, e.g., predictability of returns, growth rates and price-dividend ratio, each of which is an interesting topic for future research but beyond the scope of this paper. Another limitation of this paper is the small number of observations. It would be better if quarterly consumption data before 1994 are available. Note that the persistent parameter of economic uncertainty is arbitrarily constrained in order to get a stationary result. With more available data in the future, one should re-estimate all parameters of the long-run processes without the constraint.

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References

Abel, A. B. (1999). Risk premia and term premia in general equilibrium. *Journal of Monetary Economics*, *43*(1), 3-33.

Bansal, R., Kiku, D., & Yaron, A. (2016). Risks for the long run: Estimation with time aggregation. *Journal of Monetary Economics*, *82*, 52-69.

Bansal, R., & Yaron, A. (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *The Journal of Finance*, *59* (4), 1481-1509.

Breeden, D. T. (1979). An intertemporal asset pricing model with stochastic consumption and investment opportunities. *Journal of Financial Economics, 7*(3), 265-296.

Campbell, J. Y. (1999). Asset prices, consumption, and the business cycle. *Handbook of macroeconomics, 1*, 1231-1303.

Campbell, J. Y., & Shiller, R. J. (1988). The dividend-price ratio and expectations of future dividends and discount factors. *Review of Financial Studies, 1*(3), 195-228.

Duangchaiyoosook, S., & Ousawat, T. (2021). Equity premium puzzle in thailand: Revisited. *UTCC International Journal of Business and Economics, 13*(1), 101-133.

Duangthong, S. (2014). Asset pricing puzzle in emerging markets: An evidence of Thailand. *Southeast Asian Journal of Economics, 2*(1), 23-50.

Epstein, L. G., & Zin, S. E. (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica, 57*(4), 937-969.

Hall, R. E. (1988). Intertemporal substitution in consumption. *Journal of Political Economy, 96*(2), 339-357.

Hansen, L. P. (1982). Large sample properties of generalized method of moments estimators. *Econometrica, 50*(4), 1029-1054.

Harnphattananusorn, S. (2014). Asset pricing in capm and ccapm: Thailand case study. *Proceedings of 52nd Kasetsart University Annual Conference: Education, Economics and Business Administration, Humanities and Social Sciences*.

Kreps, D. M., & Porteus, E. L. (1978). Temporal resolution of uncertainty and dynamic choice theory. *Econometrica, 46*(1), 185-200.

Lucas, R. E. (1978). Asset prices in an exchange economy. *Econometrica, 46*(6) 1429-1445.

Mehra, R., & Prescott, E. C. (1985). The equity premium: A puzzle. Journal of Monetary Economics, 15(2), 145-161.

Newey, W. K., & West, K. D. (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica*, 55(3), 703-708.

Ronapat, M., & Evans, M. (2005). Disappearing dividends in the Thai capital market: changing firm characteristics or lower propensity to pay? *Journal of Economic & Social Policy, 10*(1), 169-212.

Sedthapinun, T. (2000). *C-capm and equity risk premium in Thailand*. (Unpublished master's thesis, Thammasat University, Bangkok, Thailand).

Weil, P. (1989). The equity premium puzzle and the risk-free rate puzzle. *Journal of Monetary Economics, 24*(3), 401-421.

Appendix

A.1 The Derivation of the Log of Price-Consumption Ratio $z_{c,t}$ as in (11)

Using log-normality of consumption growth, $G_{c,t+1}$, and return on the complete-markets portfolio, $R_{c,t+1}$, the Euler equation (4) can be rewritten in log-form as follows

$$\log\beta - \frac{1}{\psi}E_t[g_{c,t+1}] + E_t[r_{c,t+1}] + \frac{\theta}{2}Var_t[r_{c,t+1} - \frac{1}{\psi}g_{c,t+1}] = 0\,, \tag{A.1}$$

Where $\theta = \frac{\psi(1-\gamma)}{\psi-1}$. Substituting (6), (8), (10) and (11) into (A.1) and rearranging the terms lead to the following equation.

$$\begin{aligned}&\left(\log\beta + \frac{\psi-1}{\psi}\mu_c + k_{c,0} + (k_{c,1}-1)A_{c,0} + (1-\nu)k_{c,1}\sigma_c^2 A_{c,2} + \frac{\theta}{2}(\varphi_\sigma k_{c,1})^2 A_{c,2}^2\right)\\&+\left(\frac{\psi-1}{\psi} + (k_{c,1}\rho-1)A_{c,1}\right)x_t + \left(\theta\left(\frac{\psi-1}{\psi}\right)^2 - 2(1-\nu k_{c,1})A_{c,2} + \theta(k_{c,1}\varphi_x)^2 A_{c,1}^2\right)\sigma_{c,t}^2 = 0.\end{aligned} \tag{A2}$$

Since condition (A.2) must hold for every value of x_t and $\sigma_{c,t}^2$, we now can conclude that

$$\frac{\psi-1}{\psi} + (k_{c,1}\rho-1)A_{c,1} = 0\,, \tag{A.3}$$

$$\theta\left(\frac{\psi-1}{\psi}\right)^2 - 2(1-\nu k_{c,1})A_{c,2} + \theta(k_{c,1}\varphi_x)^2 A_{c,1}^2 = 0\,, \tag{A.4}$$

$$\log\beta+\frac{\psi-1}{\psi}\mu_c+k_{c,0}+(k_{c,1}-1)A_{c,0}+(1-\nu)k_{c,1}\sigma_c^2A_{c,2}+\frac{\theta}{2}(\varphi_\sigma k_{c,1}A_{c,2})^2=0, \quad \text{(A.5)}$$

We then can find the solution for $A_{c,0}$, $A_{c,1}$ and $A_{c,2}$ by solving this system of equations (A.3)- (A.5) as follows.

$$A_{c,1}=\frac{\psi-1}{\psi(1-k_{c,1}\rho)}, \quad \text{(A.6)}$$

$$A_{c,2}=\frac{\theta(\psi-1)^2+\theta(\psi k_{c,1}\varphi_x A_{c,1})^2}{2\psi^2(1-\nu k_{c,1})}, \quad \text{(A.7)}$$

$$A_{c,0}=\frac{2\psi\log\beta+2\psi k_{c,0}+2(\psi-1)\mu_c+2\psi(1-\nu)k_{c,1}\sigma_c^2A_{c,2}+\theta\psi(\varphi_\sigma k_{c,1}A_{c,2})^2}{2\psi(1-k_{c,1})}, \quad \text{(A8)}$$

which are exactly identical to the ones in Bansal and Yaron (2004). These solutions also help verify that we have guessed the process of price-consumption ratio, (11), correctly.

A.2 The Derivation of the Log of Price-Dividend Ratio $z_{m,t}$ as in (13)

Using log-normality of consumption growth, $G_{c,t+1}$, return on the complete-markets portfolio, $R_{c,t+1}$, and return on the financial market, $R_{m,t+1}$, the Euler equation (4) can be rewritten in log-form as follows.

$$\theta\log\beta-\frac{\theta}{\psi}E_t[g_{c,t+1}]+(\theta-1)E_t[r_{c,t+1}]+E_t[r_{m,t+1}]+\frac{1}{2}Var_t[(\theta-1)r_{c,t+1}+r_{m,t+1}-\frac{\theta}{\psi}g_{c,t+1}]=0, \quad \text{(B.1)}$$

where $\theta=\frac{\psi(1-\gamma)}{\psi-1}$. Substituting (6), (7), (8), (10), (11), (12) and (13) into (B.1) and rearranging the terms lead to the following equation.

$$\begin{aligned}&\log\beta+k_{m,0}-\frac{1}{\psi}\mu_c+\mu_d+\frac{1}{2}(1-\theta)\theta k_{c,1}^2A_{c,2}^2\varphi_\sigma^2+k_{m,1}A_{m,2}(1-\nu)\sigma_c^2+(1-k_{m,1})A_{m,0}\\&+\frac{1}{2}\left(k_{m,1}A_{m,2}\varphi_\sigma+(\theta-1)k_{c,1}A_{c,2}\varphi_\sigma\right)^2+\left(\frac{\phi\psi-1}{\psi}+(\rho k_{m,1}-1)A_{m,1}\right)x_t\\&+\left((1-\theta)(1-\nu k_{c,1})A_{c,2}-(1-\nu k_{m,1})A_{m,2}+\frac{\varphi_d^2+(\pi_d-\gamma)^2+(\varphi_x k_{m,1}A_{m,1}-\varphi_x(1-\theta)k_{c,1}A_{c,1})^2}{2}\right)\sigma_{c,t}^2=0.\end{aligned} \tag{B.2}$$

Since condition (B.2) must hold for every value of x_t and $\sigma_{c,t}^2$, we now can conclude that

$$\frac{\phi\psi-1}{\psi}+(\rho k_{m,1}-1)A_{m,1}=0\,, \tag{B.3}$$

$$(1-\theta)(1-\nu k_{c,1})A_{c,2}-(1-\nu k_{m,1})A_{m,2}+\frac{\varphi_d^2+(\pi_d-\gamma)^2+(\varphi_x k_{m,1}A_{m,1}-\varphi_x(1-\theta)k_{c,1}A_{c,1})^2}{2}=0\,, \tag{B.4}$$

$$\begin{aligned}&\log\beta+k_{m,0}-\frac{1}{\psi}\mu_c+\mu_d+\frac{1}{2}(1-\theta)\theta k_{c,1}^2A_{c,2}^2\varphi_\sigma^2\\&\quad+(1-k_{m,1})A_{m,0}+k_{m,1}A_{m,2}(1-\nu)\sigma_c^2+\frac{1}{2}\left(k_{m,1}A_{m,2}\varphi_\sigma+(\theta-1)k_{c,1}A_{c,2}\varphi_\sigma\right)^2=0.\end{aligned} \tag{B.5}$$

We then can find the solution for $A_{m,0}$, $A_{m,1}$ and $A_{m,2}$ by solving this system of equations (B.3)-(B.5) as follows.

$$A_{m,1}=\frac{\phi\psi-1}{\psi(1-k_{m,1}\rho)}\,, \tag{B.6}$$

$$A_{m,2}=\frac{(\pi_d-\gamma)^2+\varphi_d^2+(\varphi_x k_{m,1}A_{m,1}-\varphi_x(1-\theta)k_{c,1}A_{c,1})^2+2(1-\theta)(1-\nu k_{c,1})A_{c,2}}{2(1-\nu k_{m,1})}\,, \tag{B.7}$$

$$\begin{aligned}A_{m,0}=&\frac{2\psi\log\beta+2\psi k_{m,0}-2\mu_c+2\psi\mu_d+\psi(1-\theta)\theta(\varphi_\sigma k_{c,1}A_{c,2})^2+2\psi(1-\nu)k_{m,1}A_{m,2}\sigma_c^2}{2\psi(1-k_{m,1})}\\&+\frac{\psi(k_{m,1}A_{m,2}\varphi_\sigma+(1-\theta)k_{c,1}A_{c,2}\varphi_\sigma)^2}{2\psi(1-k_{m,1})}.\end{aligned} \tag{B.8}$$

These solutions also help verify that we have guessed the process of price-dividend ratio, (13), correctly.

A.3 The Derivation of the Expected Equity Premium $E[r_{m,t+1}-r_{f,t+1}]$ as in (14)

Using log-normality of stochastic discount factor, m_{t+1} and the financial market return, $r_{m,t+1}$, the Euler equation (4) can be rewritten in log-form as follows.

$$E_t[m_{t+1}]+E_t[r_{m,t+1}]+\frac{1}{2}Var_t[m_{t+1}]+\frac{1}{2}Var_t[r_{m,t+1}]+Cov_t[m_{t+1},r_{m,t+1}]=0. \quad \text{(C1)}$$

By construction, the Euler equation for risk-free return, $r_{f,t+1}$, can be rewritten in log-form as follows.

$$E_t[r_{f,t+1}]=-E_t[m_{t+1}]-\frac{1}{2}Var_t[m_{t+1}]. \quad \text{(C.2)}$$

Substituting (C.2) into (C.1) gives the conditional mean of the equity premium

$$E_t[r_{m,t+1}-r_{f,t+1}]=-Cov_t[m_{t+1},r_{m,t+1}]-\frac{1}{2}Var_t[r_{m,t+1}]. \quad \text{(C.3)}$$

Using (5), (6), (7), (8), (9), (10), (11), (12) and (13) and rearranging the terms, we can derive the conditional covariance between stochastic discount factor and financial market returns as follows:

$$Cov_t[m_{t+1},r_{m,t+1}]=-(\gamma)(\pi_d\sigma_{c,t}^2)-\left(\frac{\psi\gamma-1}{\psi-1}k_{c,1}A_{c,1}\varphi_x\right)(k_{m,1}A_{m,1}\varphi_x\sigma_{c,t}^2)-\left(\frac{\psi\gamma-1}{\psi-1}k_{c,1}A_{c,2}\right)(k_{m,1}A_{m,2}\varphi_\sigma^2). \quad \text{(C.4)}$$

Similarly, we can use (7), (8), (9), (12) and (13) to derive the conditional variance of financial market returns as follows:

$$Var_t[r_{m,t+1}] = Var_t\Big[k_{m,1}A_{m,1}\varphi_x\sigma_{c,t}\eta_{x,t+1} + k_{m,1}A_{m,2}\varphi_\sigma\eta_{\sigma,t+1} + \pi_d\sigma_{c,t}\eta_{c,t+1} + \varphi_d\sigma_{c,t}\eta_{d,t},$$

$$Var_t[r_{m,t+1}] = \left(k_{m,1}A_{m,1}\varphi_x\sigma_{c,t}\right)^2 + \left(k_{m,1}A_{m,2}\varphi_\sigma\right)^2 + \left(\pi_d\sigma_{c,t}\right)^2 + \left(\varphi_d\sigma_{c,t}\right)^2. \tag{C.5}$$

Using (C.3), (C.4) and (C.5), the unconditional mean of equity premium is

$$E\left[r_{m,t+1} - r_{f,t+1} \right] = \lambda_c\beta_c + \lambda_x\beta_x + \lambda_\sigma\beta_\sigma - \frac{1}{2}E\left[Var_t\left[r_{m,t+1} \right]\right], \tag{C.6}$$

where

$$E\left[Var_t\left[r_{m,t+1} \right]\right] = (k_{m,1}A_{m,1}\varphi_x\sigma_c)^2 + (\pi_d\sigma_c)^2 + (\varphi_d\sigma_c)^2 + (k_{m,1}A_{m,2}\varphi_\sigma)^2, \tag{C.7}$$

$\beta_c = \pi_d\sigma_c^2$ and $\lambda_c = \gamma$ denote the short-run risk (or consumption risk) and its price, $\beta_x = \varphi_x\sigma_c^2 k_{m,1}A_{m,1}$ and $\lambda_x = \dfrac{\varphi_x(\psi\gamma - 1)k_{c,1}}{\psi(1-\rho k_{c,1})}$ denote the long-run risk and its price, and $\beta_\sigma = \varphi_\sigma^2 k_{m,1}A_{m,2}$ and $\lambda_\sigma = \dfrac{\psi\gamma - 1}{\psi - 1}k_{c,1}A_{c,2}$ denote the volatility risk and its price.

A.4 The Derivation of the Expected risk-free return $E[r_{f,t+1}]$ as in (16)

Using log-normality of stochastic discount factor, m_{t+1}, risk-free return, $r_{m,t+1}$, can be rewritten as follows:

$$E_t[r_{f,t+1}] = -E_t[m_{t+1}] - \frac{1}{2}Var_t[m_{t+1}], \tag{D.1}$$

Following Bansal and Yaron (2004), we substitute log of stochastic discount factor (5) in the first term of (D.1), which can be written as follows:

$$r_{f,t+1} = -\log\beta + \frac{1}{\psi}E_t\left[g_{c,t+1} \right] + \frac{1-\theta}{\theta}E_t\left[r_{c,t+1} - r_{f,t+1} \right] - \frac{1}{2\theta}Var_t\left[m_{t+1} \right], \tag{D.2}$$

where $\theta = \frac{\psi(1-\gamma)}{\psi-1}$. The unconditional expectation of log of risk-free return is

$$E\left[r_{f,t+1}\right] = -\log\beta + \frac{1}{\psi}E\left[g_{c,t+1}\right] + \frac{1-\theta}{\theta}E\left[r_{c,t+1} - r_{f,t+1}\right] - \frac{1}{2\theta}E\left[Var_t\left[m_{t+1}\right]\right]. \quad \text{(D.3)}$$

It is obvious that $E\left[g_{c,t+1}\right] = \mu_c$. Similarly to the derivation in appendix C, $E\left[r_{c,t+1} - r_{f,t+1}\right]$ can be rewritten as

$$E_t[r_{c,t+1} - r_{f,t+1}] = -Cov_t[m_{t+1}, r_{c,t+1}] - \frac{1}{2}Var_t[r_{c,t+1}]. \quad \text{(D.4)}$$

Using (5), (6), (8), (9), (10) and (11), we can derive the conditional covariance between stochastic discount factor and complete-market returns as follows.

$$Cov_t[m_{t+1}, r_{c,t+1}] = -(\gamma)\left(\pi_d\sigma_{c,t}^2\right) - \left(\frac{\psi\gamma-1}{\psi-1}k_{c,1}A_{c,1}\varphi_x\right)\left(k_{c,1}A_{c,1}\varphi_x\sigma_{c,t}^2\right) - \left(\frac{\psi\gamma-1}{\psi-1}k_{c,1}A_{c,2}\right)\left(k_{c,1}A_{c,2}\varphi_\sigma^2\right). \quad \text{(D.5)}$$

Similarly, we can use (7), (8), (9), (10) and (11) to derive the conditional variance of complete-market returns as follows.

$$Var_t[r_{c,t+1}] = \left(k_{c,1}A_{c,1}\varphi_x\sigma_{c,t}\right)^2 + \left(k_{c,1}A_{c,2}\varphi_\sigma\right)^2 + \left(\sigma_{c,t}\right)^2. \quad \text{(D.6)}$$

Using (D.4), (D.5) and (D.6), we have

$$E\left[r_{c,t+1} - r_{f,t+1}\right] = \lambda_c\sigma_c^2 + \lambda_x k_{c,1}A_{c,1}\varphi_x\sigma_c^2 + \lambda_\sigma k_{c,1}A_{c,2}\varphi_\sigma^2 - \frac{1}{2}E\left[Var_t\left[r_{c,t+1}\right]\right], \quad \text{(D.7)}$$

where

$$E\left[Var_t\left[r_{c,t+1}\right]\right] = (1 + (k_{c,1}A_{c,1}\varphi_x)^2)\sigma_c^2 + (k_{c,1}A_{c,2})^2\varphi_\sigma^2. \quad \text{(D.8)}$$

We can use (5), (6), (8), (9), (10) and (11) to derive the fourth of (D.3) as follows

$$E\Big[\,Var_t\big[m_{t+1}\big]\Big]=\gamma^2\sigma_c^2+\left(\frac{1-\gamma\psi}{\psi-1}k_{c,1}A_{c,1}\varphi_x\right)^2\sigma_c^2+\left(\frac{1-\gamma\psi}{\psi-1}k_{c,1}A_{c,2}\right)^2\varphi_\sigma^2. \qquad \text{(D.9)}$$

Using (D.3), (D.7) and (D.9) , we can now derive the unconditional expectation of log of risk-free return as follows

$$E\Big[\,r_{f,t+1}\Big]=-\log\beta+\frac{\mu_c}{\psi}+\frac{\psi\gamma-1}{\psi(1-\gamma)}E\Big[\,r_{c,t+1}-r_{f,t+1}\Big]+\frac{\psi\gamma-1}{2\psi(1-\gamma)}E\Big[\,Var_t\big[m_{t+1}\big]\Big], \qquad \text{(D.10)}$$

where

$$E\Big[\,r_{c,t+1}-r_{f,t+1}\Big]=\lambda_c\sigma_c^2+\lambda_x k_{c,1}A_{c,1}\varphi_x\sigma_c^2+\lambda_\sigma k_{c,1}A_{c,2}\varphi_\sigma^2-\frac{1}{2}E\Big[\,Var_t\Big[\,r_{c,t+1}\Big]\Big], \qquad \text{(D.11)}$$

$$E\Big[\,Var_t\Big[\,r_{c,t+1}\Big]\Big]=(1+(k_{c,1}A_{c,1}\varphi_x)^2)\sigma_c^2+(k_{c,1}A_{c,2})^2\varphi_\sigma^2\,, \qquad \text{(D.12)}$$

$$E\Big[\,Var_t\big[m_{t+1}\big]\Big]=\lambda_c^2\sigma_c^2+\lambda_x^2\sigma_c^2+\lambda_\sigma^2\varphi_\sigma^2. \qquad \text{(D.13)}$$

A.5 Moment Conditions for the Long Run Processes

This appendix presents all 12 moment conditions, $f_i(\upsilon,\Theta)$, for $i=1,2,...,12$

$$f_1(\upsilon,\Theta)=g_{c,t}-\mu_c\,, \qquad \text{(E.1)}$$

$$f_2(\upsilon,\Theta)=g_{c,t}^2-\mu_c^2-\sigma_c^2-\frac{\varphi_x^2\sigma_c^2}{1-\rho^2}\,, \qquad \text{(E.2)}$$

$$f_3(\upsilon,\Theta)=g_{c,t}g_{c,t+1}-\mu_c^2-\frac{\rho\varphi_x^2\sigma_c^2}{1-\rho^2}\,, \qquad \text{(E.3)}$$

$$f_4(\boldsymbol{\upsilon},\Theta) = g_{c,t} g_{c,t+2} - \mu_c^2 - \frac{\rho^2 \varphi_x^2 \sigma_c^2}{1-\rho^2}, \tag{E.4}$$

$$f_5(\boldsymbol{\upsilon},\Theta) = g_{d,t} - \mu_d, \tag{E.5}$$

$$f_6(\boldsymbol{\upsilon},\Theta) = g_{d,t}^2 - \mu_d^2 - \pi_d^2 \sigma_c^2 - \varphi_d^2 \sigma_c^2 - \frac{\phi^2 \varphi_x^2 \sigma_c^2}{1-\rho^2}, \tag{E.6}$$

$$f_7(\boldsymbol{\upsilon},\Theta) = g_{d,t} g_{d,t+1} - \mu_d^2 - \frac{\rho \phi^2 \varphi_x^2 \sigma_c^2}{1-\rho^2}, \tag{E.7}$$

$$f_8(\boldsymbol{\upsilon},\Theta) = g_{d,t} g_{d,t+2} - \mu_d^2 - \frac{\rho^2 \phi^2 \varphi_x^2 \sigma_c^2}{1-\rho^2}, \tag{E.8}$$

$$f_9(\boldsymbol{\upsilon},\Theta) = g_{c,t} g_{d,t} - \mu_c \mu_d - \pi_d \sigma_c^2 - \frac{\phi \varphi_x^2 \sigma_c^2}{1-\rho^2}, \tag{E.9}$$

$$f_{10}(\boldsymbol{\upsilon},\Theta) = g_{c,t} g_{d,t+1} - \mu_c \mu_d - \frac{\rho \phi \varphi_x^2 \sigma_c^2}{1-\rho^2}, \tag{E.10}$$

$$\begin{aligned} f_{11}(\boldsymbol{\upsilon},\Theta) = g_{c,t}^4 - \mu_c^4 - 6\mu_c^2\sigma_c^2 - 3\sigma_c^4 - \frac{6\varphi_x^2(\mu_c^2\sigma_c^2 + \sigma_c^4)}{1-\rho^2} - \frac{3\varphi_\sigma^2}{1-\nu^2} - \frac{3\varphi_x^4\sigma_c^4}{(1-\rho^2)^2} \\ - \frac{6\nu\varphi_x^2\varphi_\sigma^2}{(1-\nu^2)(1-\nu\rho^2)} - \frac{3\varphi_x^4\varphi_\sigma^2(1+\nu\rho^2)}{(1-\nu^2)(1-\nu\rho^2)(1-\rho^4)}, \end{aligned} \tag{E.11}$$

$$
\begin{aligned}
f_{12}(\upsilon,\Theta) = {} & g_{c,t}^2 g_{d,t}^2 - \mu_c^2\mu_d^2 - (\mu_c^2\phi^2 + \mu_d^4 + 4\mu_c\mu_d\phi)\frac{\varphi_x^2\sigma_c^2}{1-\rho^2} \\
& - (\mu_c^2\pi_d^2 + \mu_c^2\varphi_d^2 + \mu_d^2 + 4\mu_c\mu_d\pi)\sigma_c^2 \\
& - (\mu_c^2\pi_d^2 + \mu_c^2\varphi_d^2 + \mu_d^2 + 4\mu_c\mu_d\pi)\sigma_c^2 - (3\pi_d^2 + \varphi_d^2)(\sigma_c^4 + \frac{\varphi_\sigma^2}{1-\nu^2}) \\
& - 3\phi^2\varphi_x^4\left(\frac{\sigma_c^4}{\left(1-\rho^2\right)^2} + \frac{\varphi_d^2(1+\nu\rho^2)}{(1-\nu^2)(1-\nu\rho^2)(1-\rho^4)}\right) \\
& - (\pi_d^2 + \varphi_d^2 + \phi^2 + 4\phi\pi_d)\left(\frac{\varphi_x^2\sigma_c^4}{1-\rho^2} + \frac{\nu\varphi_x^2\varphi_\sigma^2}{(1-\nu^2)(1-\nu\rho^2)}\right).
\end{aligned} \tag{E.12}
$$

A.6 The Relationship of Year-on-Year Returns and Growth Rates

This paper uses year-on-year calculation for both dividend and consumption growth rates to adjust for potential seasonal effects. This appendix shows that, to be consistent, we need to calculate returns using year-on-year method as well. To be concise, we show the derivation for the financial market returns and dividend growth only. Other cases can be derived readily using a similar method.

Let $P_{t,q}$ and $D_{t,q}$ be the price and dividend of an asset in quarter q of year t. The year-on-year financial market returns in quarter q of year $t+1$ are

$$R_{t+1,q} = \frac{P_{t+1,q} + D_{t+1,q}}{P_{t,q}}. \tag{F.1}$$

We can rearrange the right hand side as follows:

$$R_{t+1,q} = \frac{1+Z_{t+1,q}}{Z_{t,q}}\frac{D_{t+1,q}}{D_{t,q}}, \tag{F.2}$$