R&D networks among suppliers and manufacturers

Article *in* Economic Modelling · January 2017

DOI: 10.1016/j.econmod.2016.09.006

2 authors:


Tat Thanh Tran
University of Surrey


Vasileios Zikos
University of the Thai Chamber of Commerce

R&D Networks among Suppliers and Manufacturers*

Tat Thanh Tran† Vasileios Zikos‡

Abstract

Empirical evidence documents that R&D networks among vertically related firms are very common. Yet there is currently no formal modeling of such networks. In this paper, we develop a model of R&D networks among manufacturers and their suppliers in order to examine which network architectures emerge in equilibrium, and what their implications are from a welfare viewpoint. Our analysis reveals that private incentives to form R&D networks align with societal ones when vertical relations are non-exclusive, but may conflict when vertical relations are exclusive. In terms of policy, stricter antitrust regulation of exclusive vertical relations may, under certain conditions, be desirable from a social viewpoint.

Keywords: R&D collaboration, networks, spillovers, suppliers, manufacturers.

JEL Classification: L13, L22, O31.

*We would like to thank Sushanta Mallick (the Editor) and three anonymous referees. The authors are also grateful to Nicole Tabasso, Paul Temple, Dusanee Kesavayuth, Guilherme Carmona, Alex Gershkov, Mario Pascoa, Jaideep Roy and Antonio Mele. Zikos acknowledges financial support from the University of the Thai Chamber Commerce under its grant scheme. The usual disclaimer applies.

†School of Banking and Finance, The National Economics University, 207 Giai Phong Road, Hai Ba Trung District, Hanoi, Vietnam.

‡*Corresponding author*: Research Institute for Policy Evaluation and Design (RIPED), and School of Economics, University of the Thai Chamber of Commerce, 126/1 Vibhavadee-Rangsit Road, Dindaeng, Bangkok, 10400, Thailand. Email: v.zikos@riped.utcc.ac.th. Tel. +66 813008044. Fax: +662 6923168.

1 Introduction

A growing body of empirical studies suggests that R&D networks are a prevalent phenomenon in high-tech sectors such as information technology and pharmaceuticals (Roijakkers and Hagedoorn, 2006; Hagedoorn, 2002). It has been argued, for instance, that R&D networks are easier to establish, administer and dissolve than equity forms of R&D collaborations (e.g. Research Joint Ventures), all of which are important factors that may partly explain the increasing popularity of R&D networks within the modern business world (Narula and Hagedoorn, 1999). Nonetheless, little is known about vertical R&D networks among manufacturers and their suppliers, although such networks are common empirically. The current paper aims to develop a model of vertical R&D networks in order to examine systematically which network architectures emerge in equilibrium, and what their implications are from a welfare viewpoint.

There are different types of R&D collaborations that may arise through the formation of inter-firm networks. More specifically, R&D links can be formed among firms who are located at the same market tier – that is, *horizontal* R&D networks. But R&D links can also be formed among vertically related firms, for example, among manufacturers and their suppliers; the set of such links is often referred to as *vertical* R&D networks. In other instances, firms may choose to maintain both horizontal and vertical links – that is, they may opt to engage in both *horizontal and vertical* R&D networks.

Table 1 provides information about empirical studies on R&D collaborations, focusing on the number and percentage of firms who engaged in different types of R&D collaborations. For example, using data from a survey of German firms, Inkmann (1997) shows that out of 374 manufacturing firms engaged in R&D collaborations, 289 firms (or 77.28 percent) had only vertical R&D links, 33 firms (or 8.82 percent) had only horizontal links, and 52 firms (or 13.9 percent) had both types of links.

Table 1: Firms engaged in R&D collaborations

Empirical studies	Type	N_0	Vertical		Horizontal		Both	
			N_0	%	N_0	%	N_0	%
Inkmann (1997)	German[1]	374	289	*77.28*	33	*8.82*	52	*13.9*
Kaiser and Licht (1998)	German[2]	680	484	*71.18*	66	*9.71*	130	*19.11*
Belderbos et al. (2004)	Dutch[3]	568	338	*59.51*	74	*13.03*	156	*27.46*
Badillo and Moreno (2012)	Spanish[4]	1732	1119	*64.61*	212	*12.24*	401	*23.15*

In an empirical study using data from 14 industries in Germany during the period 1991-1993, Harabi (1998) shows that a staggering 84% of innovating firms participated in *vertical* R&D collaborations with their customers and/or suppliers. Moreover, the sharing of R&D knowledge among vertically related firms is in most cases done through non-equity forms of R&D collaboration; while equity forms have been found to account for less than 20% of the total number of R&D collaborations (Caloghirou et al., 2003).

Overall, the main findings from existing empirical studies on R&D collaboration are as follows. First, vertical R&D links (or collaborations) between firms appear to be much more common than horizontal R&D links. Second, the number of firms that maintain both vertical and horizontal R&D links is relatively smaller compared with the number of firms having vertical R&D links only. And third, vertically related firms typically prefer to share their R&D outcomes through non-equity types of R&D collaborations (such as R&D networks) rather than equity forms (such as RJVs).

Somewhat surprisingly, there is currently no formal modeling of vertical R&D networks, as previous studies have focused exclusively on horizontal R&D networks. To advance our understanding, new research is needed. In light of the aforementioned em-

[1] The first wave of the Mannheim Innovation Panel (MIP) was collected in 1993 by the Centre for European Economic Research in Mannheim on behalf of the German Ministry of Education, Research and Technology.

[2] The first five waves of the MIP were collected by ZEW and Infas–Sozialforschung on behalf of the German Ministry of Education, Research, Science and Technology.

[3] The CIS surveys in the Netherlands in 1996 and 1998.

[4] The Technological Innovation Panel (PITEC), a comprehensive database of Spanish firms collected over the period 2003-2009.

pirical evidence, new research should consider the network formation decisions in a vertically related industry. It would also be interesting to examine R&D networks among manufacturers and their suppliers, which are empirically common as explained earlier.

The current paper aims to accomplish these objectives by being the first of its kind to develop a model of vertical R&D networks among manufacturers and their suppliers. As past empirical studies suggest that the number of firms that have both types of R&D links – horizontal and vertical – or only horizontal links is much smaller than the number of firms that have *only* vertical R&D links, we focus our attention on vertical R&D collaborations.

We envisage an industry with two upstream and two downstream firms. These vertically related firms maintain either exclusive or non-exclusive relations with one another. Within this setting, we examine the endogenous formation of vertical R&D networks, that is, networks among manufacturers and their suppliers. Through the formation of such networks, manufacturers and suppliers can share know-how emanating from cost-reducing R&D investments. Our analysis reveals that, when vertical relations are non-exclusive, the complete network (in which all firms are connected) is uniquely stable and maximizes social welfare. By contrast, when vertical relations are exclusive, different network architectures emerge in equilibrium as the spillover differential between manufacturers and their suppliers varies. Yet only one network architecture maximizes welfare. Thus private incentives to form R&D networks align with societal ones in some cases but conflict in others, potentially providing new insights into the formation of R&D networks in vertically related industries.

2 Background literature and contribution

The current paper contributes to the literature on R&D cooperation among vertically related firms in oligopoly (see e.g. Banerjee and Lin, 2001; Atallah, 2002; Ishii, 2004). The approach adopted in this paper, nonetheless, differs from earlier studies on R&D

cartels and Research Joint Ventures (RJVs) in two important ways.[5] First, in an RJV or an R&D cartel firms join R&D efforts to maximize their joint profits. By contrast, in the context of R&D networks, firms choose their R&D efforts non-cooperatively (to maximize their individual profits), and subsequently communicate their R&D outcomes through spillovers. Furthermore, unlike an RJV, in an R&D network a firm can form a new R&D collaboration without the need of consent from its existing partners. Most importantly, in real world industries, R&D networks are much more common than RJVs and have been found to account for more than 80% of the total number of R&D collaborations (Caloghirou et al., 2003).

Besides the general literature on R&D cooperation, the current paper also contributes to the growing literature on R&D networks. Previous studies have mainly focused on R&D networks in one-tier industries, aiming to provide a thorough understanding of which network architectures emerge in equilibrium, and what their implications are from a welfare viewpoint (e.g. Goyal and Moraga-González, 2001; Deroian and Gannon, 2006; Zikos, 2010; Zu et al., 2011; Zirulia, 2012; Marinucci, 2014; Vonortas and Zirulia, 2015).

Goyal and Moraga-González (2001) were the first to study the endogenous formation of R&D networks among firms investing in cost-reducing R&D. Within a three-firm industry, they showed that the complete network always emerges in equilibrium, while the partial network that includes two firms but excludes the third emerges if spillovers are sufficiently low. Moreover, the number of welfare-maximizing links tends to decrease with the level of spillovers: the partial network, followed by the empty network, maximizes welfare. It appears then that private and social incentives to form R&D networks align if spillovers are sufficiently low and conflict otherwise.[6]

[5] For earlier studies on Research Joint Ventures (RJVs) and R&D cartels between firms located at the same market tier, see, for example, d' Aspremont and Jacquemin (1988); Kamien et al. (1992); Poyago-Theotoky (1995, 1999); Atallah (2005); Falvey et al. (2013); Manasakis et al. (2014); Ouchida and Goto (2016). Caloghirou et al. (2003) and Marinucci (2012) provide extensive reviews of the literature on R&D cooperation.

[6] Goyal and Moraga-González (2001) also showed that when firms compete in a homogeneous-product oligopoly, the complete network in which all firms are connected emerges in equilibrium, although it is not efficient. When firms operate in independent markets, however, the complete network is uniquely stable and efficient. These results hold under the assumption of symmetric networks in which every firm has the same number of links.

While these findings have contributed considerably to our understanding of R&D networks in one-tier industries, relatively little is known about R&D networks in two-tier industries. In this paper we extend the analysis *vertically* in order to consider the role of upstream suppliers to the downstream firms. Firms do not generally produce their own inputs but they source them from other firms. Modeling the role of input suppliers thus seems important in that respect. Moreover, like Goyal and Moraga-González (2001), we study firms' incentives to form R&D networks but shift the focus from horizontal to vertical R&D collaborations. Goyal and Moraga-González (2001) examine the formation of R&D networks among firms located at the same market tier, but they did not consider vertical links as we do in this paper. Although both types of links – horizontal and vertical – are common in real-world industries, and in some cases firms may want to establish both types of links, the formation of vertical links itself appears to be a much more prevalent phenomenon. Inkmann (1997), Kaiser and Licht (1998), Belderbos et al. (2004) and Badilo and Moreno (2012) all conclude that vertical R&D links accounted for more than 50% of the total number of links among German, Dutch and Spanish firms. What this implies is that extending the standard one-tier setting by considering the role of input suppliers and their incentives to establish R&D collaborations with final good manufacturers is important both from a theoretical and practical point of view.

Only a handful of studies in economics have examined R&D networks within the context of a vertically related industry. Kesavayuth and Zikos (2012) investigated how horizontal R&D networks emerge at the upstream and downstream market tier, but they did not consider R&D links between the two market tiers. Likewise, Kesavayuth et al. (2016) studied the formation of R&D networks among upstream firms only. Given that previous studies have focused exclusively on horizontal R&D networks, relatively little is known about vertical R&D networks, although such networks are common empirically. The current paper aims to fill this research void by developing a model of vertical R&D collaborations, among manufacturers and their suppliers, in order to examine systematically which network architectures emerge in equilibrium and what their implications are from a welfare viewpoint.

3 Model

Consider a two-tier industry consisting of two suppliers and two manufacturers denoted by S_i and M_i, $i = 1, 2$, respectively. The timing in the model is as follows. In the *first stage*, the suppliers and the manufacturers choose simultaneously their vertical R&D links. In the *second stage*, conditional on the network structure, the suppliers and the manufacturers decide simultaneously their individual R&D investments to maximize their own profits. In the *third stage*, the suppliers choose simultaneously their quantities. In the *last stage*, the manufacturers choose their output levels. This sequencing of moves is common in the R&D network literature and reflects the relative degree of commitment of longer-term vis-à-vis shorter-term decisions.

Within the current setting, we consider two distinct cases. First, we examine vertical R&D collaborations among firms that are locked in *exclusive relations*. As well as being a common assumption in the literature (as also noted by other authors who used the same assumption, for example, Milliou and Petrakis, 2007), exclusive relations may often arise between car manufacturers and their suppliers (Brenkers and Verboven, 2006).[7] Given an exclusive relation between S_i and M_i, let each supplier produce a quantity q_i of an input which is purchased at a wholesale price w_i by the respective manufacturer in order to produce a final good.

In the second case considered in this paper, we relax the assumption of exclusive relations between suppliers and manufacturers. Accordingly, we examine the effects of non-exclusive relations on the formation of vertical R&D networks. Here, each manufacturer can freely select its suppliers, while each supplier can contract with several manufacturers. Within this setting, let the suppliers produce quantity z_i of a homogeneous input which is purchased at price w by the manufacturers in order to produce a final good.

The manufacturers operate under constant returns to scale technologies, transforming

[7]Stuckey and White (1993) note that components generally require high investments in R&D. This tends to make both car manufacturers and their component suppliers vulnerable to opportunistic recontracting, which may arise if, for instance, a model turns out to be a surprising success or failure. To reduce such risk, car manufacturers may decide to enter an exclusive relation with their suppliers.

one unit of input into one unit of output. The output is subsequently sold in the product market subject to the demand function $p(Q) = a - Q$, where Q is the total output ($Q = \sum_{i=1}^{2} q_i$) and p is the market price. Suppliers and manufacturers face initial marginal costs $\bar{u} > 0$ and $\bar{d} > 0$, respectively. And by investing amounts γs_i^2 and γm_i^2, $\gamma > 0$, in cost-reducing R&D, they can attain unit production costs $\bar{u} - s_i$ and $\bar{d} - m_i$, where s_i and m_i denotes a supplier's and a manufacturer's R&D output, respectively.[8]

Suppliers and manufacturers can form vertical links in order to reduce further their marginal costs by pooling their R&D outputs. A collection of such links defines a network of vertical R&D collaborations. A natural way to examine which network architectures emerge endogenously is to consider the concept of pairwise stability proposed by Jackson and Wolinsky (1996). According to this concept, a network is pairwise stable if no firm wants to delete one of its links, and no pair of firms want to add a new link between them (with one benefiting strictly and the other at least weakly). This definition suggests that the formation of a new link requires consent of two firms – here, a manufacturer and a supplier – thus implying that a link cannot be enforced. However, a link can be simply deleted unilaterally by either of the two firms.

Following a large body of empirical studies suggesting that spillover rates may differ between market tiers (e.g. Harabi, 1998), we assume that, within a given network, spillover effects are *bi-directional* – from suppliers (manufacturers) to manufacturers (suppliers) at a rate $\theta \in (0, 1]$ ($\delta \in (0, 1]$).[9] Given that we focus our attention on within-network spillovers and to keep the analysis tractable, we assume that there are no spillover effects between firms without an R&D link between them (public spillovers); see e.g. Ferrett and Zikos (2013).[10] Hence, S_i's and M_i's marginal costs are, respectively, given

[8] When $\gamma \geqslant 2$, all equilibrium variables are non-negative and profit functions are concave. Note that numerical simulations (available on request) suggest that our key results hold for all $\gamma \geqslant 2$. Thus, for ease of illustration of our key findings and to keep the analysis tractable, we set γ equal to the lower bound value of 2 (see e.g. Goyal and Moraga-González, 2001; Kesavayuth et al., 2016).

[9] This difference in the extent of informational flows between market tiers may reflect a variety of factors, including differences in the usefulness of R&D knowledge between suppliers and manufacturers, differences in absorptive capacities, as well as differences in the efficiency of communication channels.

[10] Qualitatively similar results are obtained when the focus is on public (rather than private) spillovers, an issue we explore further in Section 6.2.

by:

$$u_i(g) = \bar{u} - s_i(g) - \delta \sum_{j \in N_i(g)} m_j(g); \tag{1}$$

and, under non-exclusive relations,

$$d_i(g) = \bar{d} + w(g) - m_i(g) - \theta \sum_{k \in M_i(g)} s_k(g), \tag{2}$$

or, under exclusive relations,

$$d_i(g) = \bar{d} + w_i(g) - m_i(g) - \theta \sum_{k \in M_i(g)} s_k(g), \tag{3}$$

where $N_i(g)$ ($M_i(g)$) denotes the set of manufacturers (suppliers) who are connected with a supplier (manufacturer) under a network g, and $a > \bar{u} + \bar{d}$. Accordingly, M_i's and S_i's profits are:

$$\pi_{M_i}(g) = [p(g) - d_i(g)]q_i(g) - \gamma[m_i(g)]^2; \tag{4}$$

and, under non-exclusive relations,

$$\pi_{S_i}(g) = [w(g) - u_i(g)]z_i(g) - \gamma[s_i(g)]^2, \tag{5}$$

or, under exclusive relations,

$$\pi_{S_i}(g) = [w_i(g) - u_i(g)]q_i(g) - \gamma[s_i(g)]^2. \tag{6}$$

It is important to note that the two manufacturers face firm-specific input prices under exclusive relations. Here, the use of firm-specific input prices works like a customer allocation agreement; each supplier has its own customer and therefore the two suppliers do not compete for the same manufacturers. This implies that the use of firm-specific input prices under exclusive relations helps to relax competition at the upstream market tier. Under non-exclusive relations, however, there is a uniform input price and this tends to increase the intensity of competition among the two suppliers. The suppliers now compete for the same manufacturers. As competition at the upstream market tier is relatively more intense when vertical relations are non-exclusive, we expect that this would affect the suppliers' profits and thus their incentives to join R&D collaborations, potentially giving rise to different R&D networks under exclusive and non-exclusive relations.

We solve the model backwards, and having determined outputs, wholesale prices and R&D investments, we turn to stage 1 in order to characterize the set of "stable" networks using the well-established equilibrium notion of pairwise stability (Jackson and Wolinsky, 1996).

4 Vertical R&D networks

The current setting allows us to study asymmetric networks, that is, networks in which firms do not necessarily maintain an equal number of links with one another. Such setting highlights the potential role of competitive advantages that firms may gain over their rivals by forming vertical R&D links. In the current setting, there are 16 possible network architectures, with 10 of them yielding qualitatively different results. Figure 1 illustrates these networks.

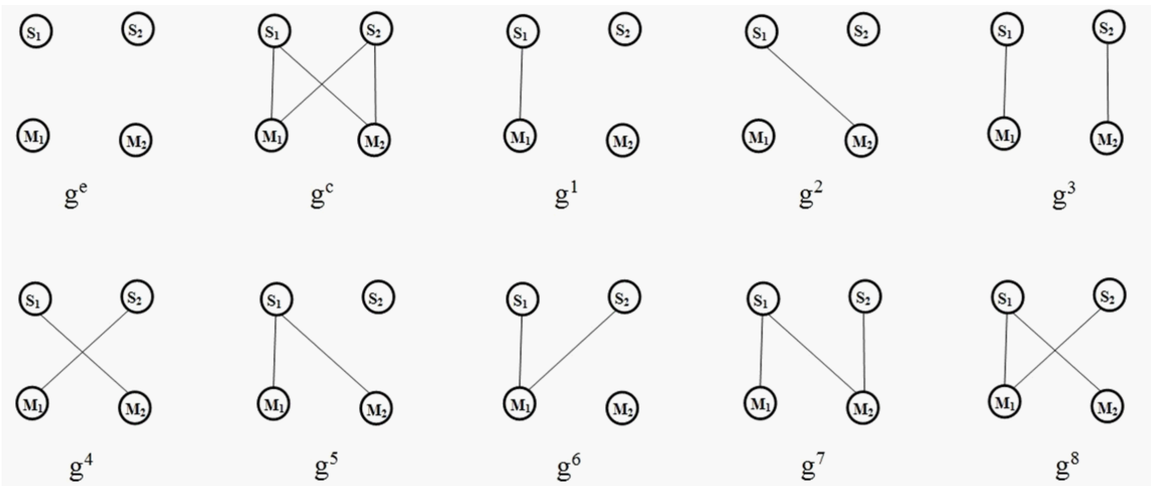

Figure 1: Vertical network architectures

4.1 Equilibrium outcomes

We now solve for the equilibrium outcomes of the different R&D networks. For practical purposes, we focus here on three key networks: g^3 and g^c under exclusive relations, and g^c under non-exclusive relations. The solutions for the other networks are given in Appendix A1.

Consider first the case where vertical relations are exclusive. In the last stage of the game, each manufacturer M_i chooses its output to maximize its profits in equation (4) given that $d_i(g^3) = \bar{d} + w_i - m_i - \theta s_i$ and $d_i(g^c) = \bar{d} + w_i - m_i - \theta(s_i + s_j)$, $i, j = 1, 2$ and $i \neq j$. The equilibrium of this stage game for g^3 and g^c respectively is:[11]

$$q_i(g^3) = (a - \bar{d} + 2m_i - m_j - 2w_i + w_j + 2s_i\theta - s_j\theta)/3$$

$$q_i(g^c) = (a - \bar{d} + 2m_i - m_j - 2w_i + w_j + (s_i + s_j)\theta)/3. \tag{7}$$

Note that under the g^3 network q_i is increasing in m_i and s_i but decreasing in m_j and s_j; while under the complete network q_i is increasing in m_i, s_i, s_j and decreasing in m_j. Inverting the systems of output demand functions above leads to:

$$w_i(g^3) = a - \bar{d} + m_i - 2q_i - q_j + s_i\theta$$

$$w_i(g^c) = a - \bar{d} + m_i - 2q_i - q_j + (s_i + s_j)\theta. \tag{8}$$

Given this, each supplier S_i chooses its quantity to maximize its profits in equation (6) given that $u_i(g^3) = \bar{u} - s_i - \delta m_i$ and $u_i(g^c) = \bar{u} - s_i - \delta(m_i + m_j)$, $i, j = 1, 2$ and $i \neq j$. The equilibrium of this stage game is:

$$q_i(g^3) = (3a_1 + 4m_i(1 + \delta) - m_j(1 + \delta) + 4s_i(1 + \theta) - s_j(1 + \theta))/15$$

$$q_i(g^c) = (3a_1 + (4 + 3\delta)m_i + (3\delta - 1)m_j + (4 + 3\theta)s_i + (3\theta - 1)s_j)/15 \tag{9}$$

[11] The second order conditions, which are always fulfilled, are available from the authors on request.

where $a_1 = a - \bar{u} - \bar{d}$ with $a > \bar{u} + \bar{d}$. In the g^3 network, q_i is increasing in m_i and s_i but decreasing in m_j and s_j. In the complete network, however, q_i is increasing in m_i and s_i; it is also increasing in m_j if $\delta > 1/3$ as well as increasing in s_j if $\theta > 1/3$. Thus, as long as spillovers are sufficiently high, S_i's and M_i's output is positively affected by R&D investments undertaken by the other firms under the complete network but not under the g^3 network.

In the preceding stage of the game, stage two, manufacturers and suppliers choose their R&D investments to maximize their profits, which yields:

$$s_i(g^3) = 4a_1(1+\theta)/B; \;\; m_i(g^3) = 2a_1(1+\delta)/B$$

$$s_i(g^c) = 2a_1(4+3\theta)/C; \;\; m_i(g^c) = a_1(4+3\delta)/C. \tag{10}$$

where $B = 69 - 2\delta(2+\delta) - 4\theta(2+\theta)$ and $C = -\delta(11+6\delta) + 2(69 - \theta(11+6\theta))$. Substitutions then show that the rest of the equilibrium outcomes for S_i's and M_i's profits and social welfare are:

$$\pi_{S_i}(g^3) = 2(a_1)^2(11-4\theta)(19+4\theta)/B^2$$

$$\pi_{M_i}(g^3) = (a_1)^2(217 - 8\delta(2+\delta))/B^2$$

$$W(g^3) = 8(a_1)^2(215 - 2\delta(2+\delta) - 8\theta(2+\theta))/B^2$$

$$\pi_{S_i}(g^c) = 8(a_1)^2(11-3\theta)(19+3\theta)/C^2$$

$$\pi_{M_i}(g^c) = 2(a_1)^2(434 - 3\delta(8+3\delta))/C^2$$

$$W(g^c) = 4(a_1)^2(-3\delta(8+3\delta) + 4(430 - 3\theta(8+3\theta)))/C^2. \tag{11}$$

Next consider the case where vertical relations are non-exclusive. In the last stage of the game, each manufacturer M_i chooses its output to maximize its profits in equation (4) given that $d_i(g^c) = \bar{d} + w - m_i - \theta(s_i + s_j)$, $i, j = 1, 2$ and $i \neq j$. The equilibrium of this stage game is:

$$q_i(g^c) = (a - \bar{d} + 2m_i - m_j - w + (s_i + s_j)\theta)/3. \tag{12}$$

Consistent with the case of exclusive relations, q_i is increasing in m_i, s_i and s_j but decreasing in m_j. We then sum up q_i to obtain the total quantity demanded in the downstream market, Q. The total output of the two manufacturers equals the total output of the two suppliers, i.e. $Q = z_1 + z_2$. Inverting the output demand function yields:

$$w(g^c) = (2(a - \bar{d}) + m_1 + m_2 - 3(z_1 + z_2) + 2(s_1 + s_2)\theta)/2.$$

Given this, each supplier S_i chooses its quantity to maximize its profits in equation (5) given that $u_i(g^c) = \bar{u} - s_i - \delta(m_i + m_j)$, $i, j = 1, 2$ and $i \neq j$. The equilibrium of this stage game is:

$$z_i(g^c) = (2a_1 + (1 + 2\delta)m_i + (1 + 2\delta)m_j + 2(2 + \theta)s_i + 2(\theta - 1)s_j)/9 \tag{13}$$

where $a_1 = a - \bar{u} - \bar{d}$ with $a > \bar{u} + \bar{d}$. Note that z_i is increasing in m_i, m_j and s_i but decreasing in s_j, indicating that the suppliers now compete for the same downstream customers (i.e. manufacturers). Thus, in the complete network, s_j always impacts z_i negatively under non-exclusive relations, but s_j may affect q_i positively under exclusive relations – provided that spillovers θ are sufficiently high. This in turn suggests that competition at the upstream market tier is more intense with non-exclusive than with exclusive relations.

In stage two of the game, manufacturers and suppliers choose their R&D investments to maximize their profits, and this leads to:

$$s_i(g^c) = 6a_1(2 + \theta)/D; \;\; m_i(g^c) = a_1(11 + 4\delta)/D. \tag{14}$$

where $D = 139 - 26\delta - 8\delta^2 - 6\theta(5 + 2\theta)$. Substitutions then reveal the rest of the equilibrium outcomes for S_i's and M_i's profits and social welfare:

$$\pi_{S_i}(g^c) = 72(a_1)^2(23 - \theta(4+\theta))/D^2$$

$$\pi_{M_i}(g^c) = 2(a_1)^2(527 - 8\delta(11+2\delta))/D^2$$

$$W(g^c) = 4(a_1)^2(2003 - 8\delta(11+2\delta) - 36\theta(4+\theta))/D^2. \tag{15}$$

4.2 Equilibrium networks

Using the equilibrium notion of pairwise stability (Jackson and Wolinsky, 1996) explained earlier, we obtain the following proposition:

Proposition 1 *In the parameter space (δ, θ):*

Under exclusive relations, two vertical R&D networks emerge in equilibrium as pairwise stable:

(i) g^3 when the difference between δ and θ is sufficiently large;

(ii) g^c when the difference between δ and θ is sufficiently small;

(iii) both g^3 and g^c when the difference between δ and θ is intermediate.

Under non-exclusive relations, g^c emerges as the unique pairwise stable network.

Proposition 1 suggests that, when vertical relations are exclusive, the equilibrium R&D networks are sensitive to the spillover differentials between manufacturers and their suppliers. What are the main effects that tend to make different networks emerge in equilibrium? Consider the network g^1 where only supplier S_i and manufacturer M_i maintain a link. Within this network, S_i and M_i enjoy a cost-saving effect relative to S_j and M_j due to their superior access to lower costs through R&D. To internalize this negative externality on their profits, S_j and M_j have an incentive to form a link with each other. In turn, forming this link destabilizes the g^1 network, and the g^3 network emerges in equilibrium.

Under exclusive relations there are two competing vertical chains. This implies that forming a link across the two chains (between S_i and M_j) generally makes M_j a more aggressive competitor to M_i – and this tends to limit S_i's sales in the input market. But if the spillover differential between S_i and M_j is relatively small, both firms benefit from a cost-saving effect, and S_i does not suffer a substantial loss in input sales. Put differently, M_j cannot 'steal' much business from M_i (business stealing effect), given that S_i and M_j enjoy a similar incoming spillover. Thus, as long as the spillover differential is sufficiently small, the cost-saving effect outweighs the business-stealing effect, and complete network emerges in equilibrium when vertical relations are exclusive, as Proposition 1 reports.

By contrast, if the spillover differential is sufficiently large, either S_i or M_j is bound to benefit relatively less from its R&D collaboration with the other firm. If S_i, for instance, receives a relatively lower spillover from M_j, this tends to make M_j a more aggressive competitor to M_i and thereby limits S_i's sales in the input market. In other words, S_i enjoys a smaller cost-saving effect relative to M_j, which enables M_j to steal business from M_i. What this implies is that S_i has now an incentive to break its link with M_j. Thus, as long as the spillover differential is sufficiently large, the business stealing effect outweighs the cost-saving effect, and the g^3 network emerges in equilibrium under exclusive relations, as Proposition 1 states. Also, for all other intermediate levels of spillover differentials, the two effects – the cost-saving effect and the business stealing effect – offset each other. Hence, g^3 and g^c are likely to be both pairwise stable. For ease of reading, these findings are better illustrated in Figure 2.[12]

[12] In Figure 2, the curves C_1, C_2, C_3 and C_4 are the sets of (δ, θ) values that solve the equations $\pi_{M_1}(g^7) = \pi_{M_1}(g^c)$, $\pi_{M_2}(g^7) = \pi_{M_2}(g^3)$, $\pi_{S_1}(g^7) = \pi_{S_1}(g^3)$ and $\pi_{S_2}(g^7) = \pi_{S_2}(g^c)$, respectively. In plotting the figure, we have normalized $a_1 = a - \bar{u} - \bar{d}$ to 1 (i.e. $a = 3$, $\bar{u} = 1$ and $\bar{d} = 1$), which is inconsequential in a qualitative sense as a_1 is a scale parameter.

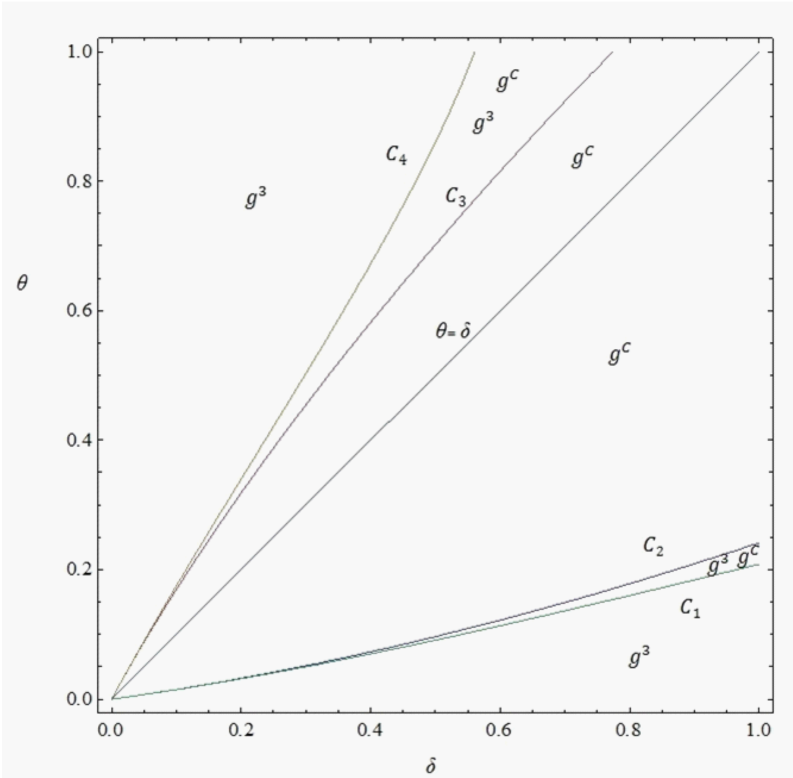

Figure 2: Pairwise stable industry structures under exclusive relations

When vertical relations are non-exclusive, only the network g^c emerges in equilibrium. In this case, there are no competing vertical chains: S_i and S_j compete for the same manufacturers. Moreover, as there are no distinct 'demand lines' for the suppliers' outputs, a link between S_i and M_i does not differ in nature from a link between S_i and M_j. What this implies is that if S_i, for instance, receives a relatively lower spillover from M_j, S_i does no longer need to be concerned about a potential negative externality on M_i's profits. Put differently, the business stealing effect that tends to reduce M_i's profits under exclusive relations would vanish altogether from S_i's viewpoint when vertical relations are non-exclusive. A link between S_i and M_j now implies only a cost-saving effect, giving thus rise to the complete network, as Proposition 1 reports.

5 Social welfare

In this section, we investigate the efficiency of vertical R&D networks. A natural question of interest is whether "market forces" governing network formation will lead to an outcome that is also desirable from a social viewpoint. To address this question, we define social welfare in a network g as the sum of consumers' surplus, suppliers' profits and manufacturers' profits:

$$W(g) = \frac{[Q(g)]^2}{2} + \sum_{i=1}^{2} \pi_{S_i}(g) + \sum_{i=1}^{2} \pi_{M_i}(g). \tag{16}$$

Our analysis reveals that, under both settings of vertical relations, the addition of a link between manufacturers and their suppliers reduces the marginal costs of both parties, thus leading to an increase in producer surplus. The resulting lower input prices and final product prices imply a higher surplus for the consumers. In consequence, the complete network that contains a larger number of links than any other network is the unique welfare-maximizing network, as Proposition 2 states.

Proposition 2 *Under exclusive and non-exclusive relations, the addition of a vertical link to an existing R&D network increases social welfare. Therefore the unique welfare-maximizing network is the complete R&D network for all values of voluntary spillovers.*

Taken together, Propositions 1 and 2 indicate that private and social incentives to form collaborative agreements are not necessarily aligned. While the complete network is always welfare-maximizing, it emerges in equilibrium only if the spillover differential between manufacturers and their suppliers is intermediate or sufficiently small under exclusive relations. In other words, when the spillover differential is sufficiently large, i.e. in the area to the left of the curve C_4 or below the curve C_1, the industry participants pursuing their private interests do not attain an outcome that is socially desirable. The equilibrium R&D network (g^3) is likely to be "under-connected" from a social viewpoint. By contrast, under non-exclusive relations, it appears that private and social incentives to form collaborative agreements are aligned. We may conclude that the potential conflict between stability and social welfare under exclusive relations (when the spillover differential is sufficiently large) does not seem to present itself under the non-exclusive relations setting.

6 Extensions

We extend our analysis in two main directions: the number of firms and public spillovers.

6.1 Networks with n firms at each tier

Consider a two-tier industry consisting of n suppliers and n manufacturers denoted by S_i and M_i, $i = 1, 2, ..., n$, respectively. As it is not analytically tractable to solve a model of asymmetric networks with an arbitrary number of firms (see Goyal and Moraga-González, 2001, for a detailed discussion), we focus our attention on symmetric networks in which every firm has the same number of links. Following a large body of empirical studies in the literature suggesting that spillover rates may differ between market tiers (e.g. Harabi, 1998), we analyze 3 distinct cases allowing us to obtain analytical solutions of a model with n firms at each market tier while also permitting comparison with our previous findings. The 3 cases we consider here are as follows: $(i) : (\theta = 1, \delta = 0)$; $(ii) : (\theta = 0, \delta = 1)$ and $(iii) : \theta = \delta = 1$.

Within the current setting, we examine two network structures. First, the network g_k^l in which every supplier S_i has an R&D link with every manufacturer M_i and k vertical links with other k manufacturers.[13] Second, the network g_k in which every S_i has no R&D link with M_i but k vertical links with other k manufacturers.

With respect to network stability, our analysis yields the following result.[14]

Proposition 3 *Under exclusive relations:*

If $\theta = \delta = 1$, the complete network g_{n-1}^l is pairwise stable while the empty network g_0 and the parallel network g_0^l are not pairwise stable.

If $(\theta = 1,\ \delta = 0)$ or $(\theta = 0,\ \delta = 1)$, the parallel network is pairwise stable while the complete network and the empty network are not pairwise stable.

[13] Note that the *parallel network* refers to the case in which each pair of firms S_i and M_i have a link while there is no link between S_i and M_j.

[14] We consider the profits of the manufacturers and the suppliers in order to identify the pairwise stable networks. The equilibrium outcomes turn out to be very lengthy and hence we are not able to present them here. Plots supporting the comparisons of equilibrium profits under the different networks are available on request.

Under non-exclusive relations:

The complete network g^l_{n-1} *is pairwise stable for all three cases* $(i) : (\theta = 1,\ \delta = 0)$*;* $(ii) : (\theta = 0,\ \delta = 1)$ *and* $(iii) : \theta = \delta = 1$*, while the empty network* g_0 *and the parallel network* g^l_0 *are not pairwise stable.*

It should be noted here that $\theta = \delta = 1$ corresponds to the case in which the difference between δ and θ is sufficiently small in our baseline model; while $(\theta = 1,\ \delta = 0)$ or $(\theta = 0,\ \delta = 1)$ correspond to the case in which the difference between δ and θ is sufficiently large. Proposition 3 thus confirms that our earlier findings reported in Proposition 1 may also be applied to networks with n firms at each market tier and voluntary spillovers at extreme values.

In terms of social welfare, given the complexity of the computations involved, we have been unable to obtain a general characterization of welfare-maximizing networks. In an attempt to circumvent this issue, we focus our attention on the network g^l_k (in which every S_i has an R&D link with every M_i and k vertical links with other k manufacturers), given that Proposition 2 suggests that the complete network is always a welfare-maximizing architecture. The following proposition summarizes our findings.[15]

Proposition 4 *Under exclusive and non-exclusive relations, social welfare increases with* k *in* g^l_k*. Therefore the complete network is the unique welfare-maximizing network within the class of networks* g^l_k*.*

The result in Proposition 4 is consistent with the one reported in Proposition 2, thus lending support for our previous findings.

Assuming symmetric networks, the analysis to this point has considered a setting with n firms at each market tier. A question of interest is whether our results continue to hold if we extend the analysis to the context of asymmetric networks. This is a difficult question to address given that our model would no longer be tractable analytically (Goyal and Moraga-González, 2001), while a simulation model does not seem able to break this

[15] A proof of this result is available from the authors on request.

barrier. Nevertheless, the simple setting employed here allows us to draw some conjectures about what might happen in a more general setting.

If our baseline model with two suppliers and two manufacturers is extended symmetrically at the two market tiers, we would expect that the key mechanism identified earlier is still at work. More specifically, when vertical relations are exclusive, the "quality" of a link between S_i and M_j would still depend on the extent of the spillover differential between them. If the spillover differential is relatively small, both firms would benefit from a cost-saving effect and neither would suffer a substantial loss in output sales. Thus, like in our original model, the cost-saving effect would likely dominate the business stealing effect, leading to a denser network structure. The intuition would work in the opposite direction when the spillover differential between S_i and M_j is sufficiently large, suggesting the formation of a sparser network.

On the other hand, when vertical relations are non-exclusive, and S_i considers forming a link with M_j, it does no longer need to be concerned about a potential negative externality on M_i's profits. What this implies is that the business stealing effect that tends to reduce M_i's profits under exclusive relations would vanish altogether from S_i's viewpoint when vertical relations are non-exclusive. Hence, consistent with our original model, a link between S_i and M_j would now imply only a cost-saving effect, potentially encouraging firms to expand their network by bringing in new members. At any rate, it is still advisable for the readers to treat our interpretations here with caution, and future research will need to return to studying vertical R&D networks in a more general setting.

6.2 Public spillovers

The analysis to this point has considered spillovers within a network. As a robustness check, we re-conduct our analysis by allowing for spillovers outside a network, i.e. public spillovers. Here, we still maintain the assumption that spillovers are bi-directional – from suppliers (manufacturers) to manufacturers (suppliers) at a rate of $\alpha \in (0, 1]$ ($\beta \in (0, 1]$). As public spillovers are typically considered to be of a smaller magnitude than private spillovers, we assume that $\alpha \leqslant \theta$ and $\beta \leqslant \delta$. S_i's and M_i's marginal costs are, respectively, given by:

$$u_i(g) = \bar{u} - s_i(g) - \delta \sum_{j \in N_i(g)} m_j(g) - \beta \sum_{h \notin N_i(g)} m_h(g); \tag{17}$$

and, under non-exclusive relations,

$$d_i(g) = \bar{d} + w(g) - m_i(g) - \theta \sum_{k \in M_i(g)} s_k(g) - \alpha \sum_{l \notin M_i(g)} s_l(g), \tag{18}$$

or, under exclusive relations,

$$d_i(g) = \bar{d} + w_i(g) - m_i(g) - \theta \sum_{k \in M_i(g)} s_k(g) - \alpha \sum_{l \notin M_i(g)} s_l(g). \tag{19}$$

Because $\alpha \leqslant \theta$ and $\beta \leqslant \delta$, we consider the case in which $\theta = \delta = 1$. Doing so allows us to study the whole range of public spillovers, i.e. $\alpha \in (0, 1]$ and $\beta \in (0, 1]$. Our approach here is similar to that adopted by Goyal and Moraga-González (2001) and Kesavayuth et al. (2016), among others. Our key findings are put forward in the following proposition.[16]

Proposition 5 *In the parameter space (α, β):*

Under exclusive relations, two vertical R&D networks emerge in equilibrium as pairwise stable:

(i) g^3 when the difference between α and β is sufficiently large;

(ii) g^c when the difference between α and β is sufficiently small;

(iii) both g^3 and g^c when the difference between α and β is intermediate.

Under non-exclusive relations, g^c emerges as the unique pairwise stable network.

Proposition 5 suggests that, when vertical relations are non-exclusive, the complete network is uniquely stable. By contrast, when vertical relations are exclusive, different network architectures emerge in equilibrium as the (public) spillover differential between manufacturers and their suppliers varies. For ease of reading, Figure 3 illustrates our key

[16]The proof of Proposition 5 is very similar to that presented for Proposition 1.

findings under the setting of exclusive relations. As we can see, Figure 3 is roughly the mirror image of Figure 2. That is, as private spillovers of a rate of θ and δ in Figure 2 get bigger, the extent of within-network information sharing between manufacturers and their suppliers increases. Indeed, for sufficiently high values of θ and δ we get close, in a qualitative sense, to the case in which α and β take on sufficiently small values in Figure 3. Therefore, it appears that the findings in the present setting are qualitatively similar, regardless of a focus on public or private spillover effects.

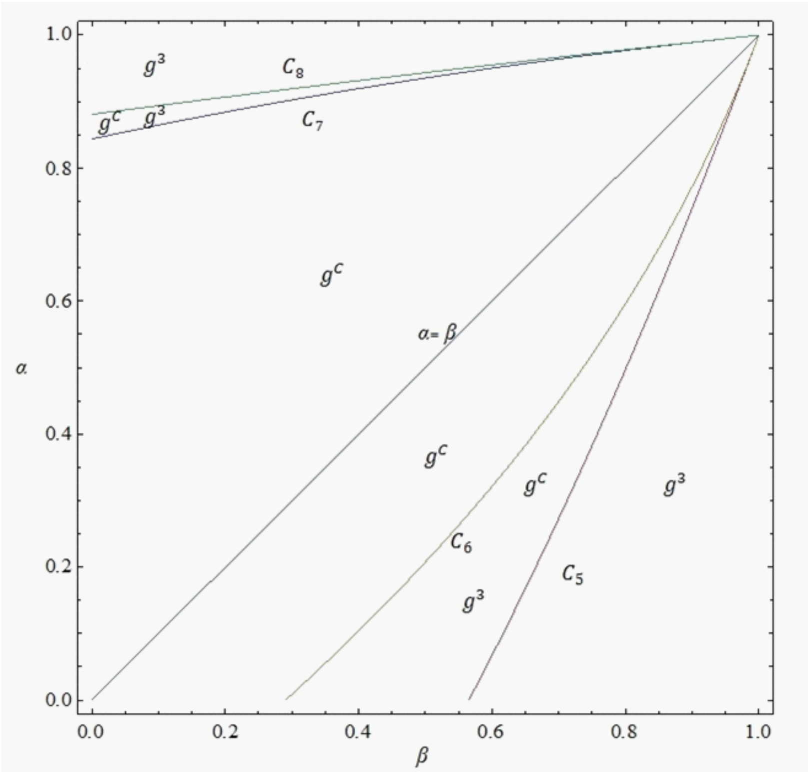

Figure 3: Pairwise stable industry structures under exclusive relations

In terms of social welfare, we obtain the following proposition.[17]

Proposition 6 *Under exclusive and non-exclusive relations, the addition of a vertical link to an existing R&D network increases social welfare. Therefore the unique welfare-maximizing network is the complete R&D network for all values of public spillovers.*

[17]The result follows directly from the comparisons: $W(g^e) < W(g^2) < W(g^4) < W(g^8) < W(g^c)$; $W(g^2) < W(g^i) < W(g^j) < W(g^c)$; $W(g^e) < W(g^1) < W(g^3) < W(g^7) < W(g^c)$; and $W(g^1) < W(g^i) < W(g^j) < W(g^c)$ for all α and $\beta \in (0, 1]$, where $i \in \{5, 6\}$ and $j \in \{7, 8\}$.

As we can see, the results reported in Proposition 6 are consistent with those in Proposition 2, thus lending further support for our earlier findings. What Proposition 6 also seems to suggest is that public spillovers within a two-tier industry may be another source of "under-connected" R&D networks under exclusive relations; that is, a source of a potential conflict between private and social incentives to form vertical R&D networks.

7 Concluding remarks

This paper provides some of the first insights into the incentives of suppliers and manufacturers to form vertical R&D networks. Building a model of endogenous network formation, we have shown that vertical R&D networks may depend not only on the nature of vertical relations but also on the extent of bi-directional spillover effects arising between manufacturers and their suppliers. More specifically, when vertical relations are non-exclusive, the complete network emerges in equilibrium and maximizes social welfare. When vertical relations are exclusive, however, we find that different network architectures emerge in equilibrium as the spillover differential between manufacturers and their suppliers varies. Yet it appears that only one architecture – the complete network – maximizes welfare. We may conclude that private and social incentives to form R&D networks always align when vertical relations are non-exclusive, but may conflict when vertical relations are exclusive, providing new insights into the stability and efficiency properties of vertical R&D networks.

What are the policy implications of these findings? Antitrust law in the U.S. and the E.U. assesses exclusive relations in a relatively lenient way, using in some cases a rule of reason analysis which aims to balance pro-competitive and anti-competitive effects (Federal Trade Commission, 2016; Galarza et al. 2012). In the current setting we have shown that when vertical relations are exclusive, the equilibrium R&D network is "under-connected" if the spillover differential between manufacturers and their suppliers is sufficiently large. Intuitively, when vertical relations are exclusive, there are two competing vertical chains and each supplier charges a specific input price to the corresponding manufacturer. Such input prices serve to relax competition at the upstream market tier, but under certain circumstances they tend to reduce overall welfare. What this implies is that stricter regulation of exclusive vertical relations may be desirable from a social viewpoint if the spillover differential between manufacturers and their suppliers is sufficiently large. By contrast, when vertical relations are non-exclusive, private incentives to form R&D networks are aligned with societal ones, implying that a laissez-faire type of policy might be preferable in that case.

Overall, our findings contribute to the literature on R&D networks by bringing some new insights into the role of specific economic factors – such as spillover effects and the nature of vertical relations – that may help us to better understand which architectures of vertical R&D networks emerge in equilibrium and what their implications are from a welfare viewpoint.

Our paper is not without shortcomings. One natural objection is that the finding that only one architecture – the complete network – emerges in equilibrium under non-exclusive relations is of limited interest. Although this appears to be a recurring result in network papers, allowing for diminishing returns to the number of links would generally require a different model. Goyal and Joshi (2003), for instance, consider a model where firms establish costly links and benefit from an exogenously specified cost reduction as a result. In that setting, asymmetric network structures turn out to be stable even among ex ante symmetric firms. Such further research might be a natural development of our model. Understanding network formation is certainly a worthwhile goal and calls for an increased research focus on investigating such networks in the context of vertically related industries.

References

[1] Atallah, G., 2002. Vertical R&D Spillovers, Cooperation, Market Structure, and Innovation. Economics of Innovation and New Technology, 11(3), 179-209.

[2] Atallah, G., 2005. R&D cooperation with asymmetric spillovers. Canadian Journal of Economics, 38(3), 919-936.

[3] Badillo, E and Moreno, R., 2012. What Drives the Choice of Partners in R&D Cooperation? Heterogeneity across Sectors. Research Institute of Applied Economics, Working Paper 2012/13.

[4] Banerjee, S. and Lin, P., 2001. Vertical research joint ventures. International Journal of Industrial Organization 19, 285-302.

[5] Belderbos, R, Carree, M, Diederen, B, Lokshin, B, Veugelers, R., 2004. Heterogeneity in R&D cooperation strategies. International Journal of Industrial Organization 22, 1237-1263.

[6] Brenkers, R. and Verboven, F., 2006. Liberalizing a distribution system: the European car market. Journal of the European Economic Association 4, 216-251.

[7] Caloghirou, Y., Ioannides, S. and Vonortas, N., 2003. Research joint ventures. Journal of Economic Surveys 17, 541-570.

[8] d' Aspremont, C. and Jacquemin, A., 1988. Cooperative and Noncooperative R&D in Duopoly with Spillovers. American Economic Review 78, 1133-37.

[9] Deroian, F. and Gannon, F., 2006. Quality-improving alliances in differentiated oligopoly. International Journal of Industrial Organization 24, 629-637.

[10] Falvey, R., Poyago-Theotoky, J. and Teerasuwannajak, K. T., 2013. Coordination costs and research joint ventures. Economic Modelling 33, 965-976.

[11] Federal Trade Commission, 2016, May 31. Exclusive Dealing or Requirements Contracts. Retrieved from https://www.ftc.gov/tips-advice/competition-guidance/guide-antitrust-laws/dealings-supply-chain/exclusive-dealing-or.

[12] Ferrett, B. and Zikos, V. (2013). Wage-Setting Institutions and R&D Collaboration Networks. Australian Economic Papers 52(2), 61-78.

[13] Galarza, A.F., Dziadykiewicz, E.L. and Figueroa, P., 2012. Exclusive distribution: An overview of EU and national case law. e-Competitions, No 41235.

[14] Goyal, S. and Joshi, S., 2003. Networks of collaboration in oligopoly. Games and Economic Behavior 43(1), 57-85.

[15] Goyal, S. and Moraga-González, J.L., 2001. R&D Networks. Rand Journal of Economics, 32, 686-707.

[16] Hagedoorn, J., 2002. Inter-firm R&D Partnerships: An Overview of Major Trends and Patterns since 1960. Research Policy 31, 477-492.

[17] Harabi, N., 1998. Innovation Through Vertical Relations Between Firms, Suppliers And Customers: A Study Of German Firms. Industry and Innovation 5(2), 157-179.

[18] Jackson, M. O. and Wolinsky, A., 1996. A Strategic Model of Social and Economic Networks. Journal of Economic Theory 71, 44-74.

[19] Inkmann, J., 1997. Horizontal and Vertical R&D Cooperation, Universität Konstanz, mimeo.

[20] Ishii, A., 2004. Cooperative R&D between vertically related firms with spillovers. International Journal of Industrial Organization 22, 1213-1235.

[21] Kaiser, U. and Licht, G., 1998. R&D cooperation and R&D intensity: Theory and Micro-Econometric Evidence for Germany, ZEW Discussion Paper No. 98-32, Mannheim.

[22] Kamien, M.I., Muller, E., and Zang, I., 1992. Research Joint Ventures and R&D Cartels. American Economic Review 85, 1293-1306.

[23] Kesavayuth, D., Manasakis, C. and Zikos, V., 2016. Endogenous R&D Networks among Upstream Firms. Managerial and Decision Economics 37, 79-94.

[24] Kesavayuth, D. and Zikos, V., 2012. Upstream and downstream horizontal R&D networks. Economic Modelling 29, 742-750.

[25] Manasakis, C., Petrakis, E. and Zikos, V., 2014. Downstream Research Joint Venture with Upstream Market Power. Southern Economic Journal 80, 782-802.

[26] Marinucci, M., 2012. A primer on R&D cooperation among firms. Bank of Italy Occasional Paper, No. 130.

[27] Marinucci, M., 2014. Cooperative R&D networks among firms and public research institutions. Bank of Italy Temi di Discussione (Working Paper) No. 962.

[28] Milliou, C. and Petrakis, E., 2007. Upstream horizontal mergers, vertical contracts, and bargaining. International Journal of Industrial Organization 25, 963-987.

[29] Narula, R. and Hagedoorn, J., 1999. Innovating through strategic alliances: moving towards international partnerships and contractual agreements. Technovation 19, 283-294.

[30] Ouchida, Y. and Goto, D., 2016. Environmental research joint ventures and time-consistent emission tax: Endogenous choice of R&D formation. Economic Modelling 55, 179-188.

[31] Poyago-Theotoky, J., 1995. Equilibrium and Optimal Size of a RJV in an Oligopoly with Spillovers. Journal of Industrial Economics 43, 209-226.

[32] Poyago-Theotoky, J., 1999. A Note on Endogenous Spillovers in a Non-Tournament R&D Duopoly. Review of Industrial Organization 15, 253-262.

[33] Roijakkers, N. and Hagedoorn, J., 2006. Inter-firm R&D partnering in pharmaceutical biotechnology since 1975: Trends, patterns, and networks. Research Policy 35, 431-446.

[34] Stuckey, J. and White, D. (1993). When and When Not to Vertically Integrate. Sloan Management Review, 34(3), 71-83.

[35] Vonortas, N. and Zirulia, L., 2015. Strategic technology alliances and networks. Economics of Innovation and New Technology, 24, 490-509.

[36] Zikos, V., 2010. R&D Collaboration Networks in Mixed Oligopoly. Southern Economic Journal 77, 189-212.

[37] Zirulia, L., 2012. The role of spillovers in R&D network formation. Economics of Innovation and New Technology 21, 83-105.

[38] Zu, L., Dong, B., Zhao, X., Zhang, J., 2011. International R&D Networks. Review of International Economics 19, 325-340.

Appendix

Appendix A1. Equilibrium Outcomes

We present the equilibrium outcomes for the different architectures of vertical R&D networks.[18] Let $a_1 = a - \bar{u} - \bar{d}$ with $a > \bar{u} + \bar{d}$. Equilibrium outcomes are as follows.

The empty network (g^e)

Exclusive relations:

$\pi_{S_i}(g^e) = 418(a_1)^2/4761$

$\pi_{M_i}(g^e) = 217(a_1)^2/4761$; $W(g^e) = 1720(a_1)^2/4761$.

Non-exclusive relations:

$\pi_{S_i}(g^e) = 1656(a_1)^2/19321$

$\pi_{M_i}(g^e) = 1054(a_1)^2/19321$; $W(g^e) = 8012(a_1)^2/19321$.

The pattern 1 network (g^1)

Exclusive relations:

$\pi_{S_1}(g^1) = 338(a_1)^2(209 - 32\theta - 16\theta^2)/9(B_1)^2$

$\pi_{S_2}(g^1) = 418(a_1)^2(39 - 4\delta - 2\delta^2 - 8\theta - 4\theta^2)^2/81(B_1)^2$

$\pi_{M_1}(g^1) = 169(a_1)^2(217 - 16\delta - 8\delta^2)/9(B_1)^2$

$\pi_{M_2}(g^1) = 217(a_1)^2(39 - 4\delta - 2\delta^2 - 8\theta - 4\theta^2)^2/81(B_1)^2$

$W(g^1) = 26(a_1)^2F_1/81(B_1)^2$

where $B_1 = 299 - 24\delta - 12\delta^2 - 48\theta - 24\theta^2$; $F_1 = 460\delta^3 + 115\delta^4 + 8\delta(115\theta^2 + 230\theta - 1407) + \delta^2(460\theta^2 + 920\theta - 5168) + 4(25155 - 60096\theta - 2588\theta^2 + 460\theta^3 + 115\theta^4)$.

Non-exclusive relations:

$\pi_{S_1}(g^1) = 2(a_1)^2(2100 + 191\delta + 50\delta^2)^2(92 - 8\theta - \theta^2)/(B_2)^2$

$\pi_{S_2}(g^1) = 184(a_1)^2[150(\theta - 14) + \delta^2(50 + 24\theta + 6\theta^2) + \delta(359 + 126\theta + 33\theta^2)]^2/(B_2)^2$

$\pi_{M_1}(g^1) = 2(a_1)^2(527 - 44\delta - 4\delta^2)(700 + 143\theta + 42\theta^2)^2/(B_2)^2$

$\pi_{M_2}(g^1) = 1054(a_1)^2[42\theta^2 + 193\theta - 700 + 2\delta^2\theta(4 + \theta) + \delta(56 + 42\theta + 11\theta^2)]^2/(B_2)^2$

where $B_2 = 25(42\theta^2 + 349\theta - 3892) + \delta^2(700 + 824\theta + 206\theta^2) + \delta(8442 + 43326\theta + 1133\theta^2)$.

[18] The second order conditions are available from the authors on request.

The pattern 2 network (g^2)

Exclusive relations:

$\pi_{S_1}(g^2) = 2(a_1)^2(78 + 5\delta - \delta^2)^2(209 + 8\theta - \theta^2)/(B_3)^2$

$\pi_{S_2}(g^2) = 1672(a_1)^2(39 + 5\theta - \theta^2)^2/(B_3)^2$

$\pi_{M_1}(g^2) = 217(a_1)^2(78 + 5\delta - \delta^2)^2/(B_3)^2$

$\pi_{M_2}(g^2) = 2(a_1)^2(434 + 8\delta - \delta^2)(39 + 5\theta - \theta^2)^2/(B_3)^2$

where $B_3 = \delta(15\theta - 114 - 4\theta^2) + \delta^2(15 - 4\theta + \theta^2) + 6(5\theta^2 - 897 - 38\theta)$.

Non-exclusive relations:

g^2 and g^1 imply the same equilibrium outcomes.

The pattern 3 network (g^3)

Exclusive relations:

The equilibrium outcomes are in Section 4.1.

Non-exclusive relations:

$\pi_{S_i}(g^3) = 18(a_1)^2(92 - 8\theta - \theta^2)/(B_4)^2$

$\pi_{M_i}(g^3) = 2(a_1)^2(527 - 44\delta - 4\delta^2)/(B_4)^2$

$W(g^3) = 4(a_1)^2(2003 - 44\delta - 4\delta^2 - 72\theta - 9\theta^2)/(B_4)^2$

where $B_4 = 139 - 13\delta - 2\delta^2 - 15\theta - 3\theta^2$.

The pattern 4 network (g^4)

Exclusive relations:

$\pi_{S_i}(g^4) = 8(a_1)^2(209 + 8\theta - \theta^2)/(B_5)^2$

$\pi_{M_i}(g^4) = 2(a_1)^2(434 + 8\delta - \delta^2)/(B_5)^2$

$W(g^4) = 4(a_1)^2(8\delta - \delta^2 + 4(430 + 8\theta - \theta^2))/(B_5)^2$

where $B_5 = 3\delta - \delta^2 - 2(69 - 3\theta + \theta^2)$.

Non-exclusive relations:

g^4 and g^3 imply the same equilibrium outcomes.

The pattern 5 network (g^5)

Exclusive relations:

$\pi_{S_1}(g^5) = 2(a_1)^2(78 + 5\delta - \delta^2)^2(209 - 24\theta - 9\theta^2)/(B_6)^2$

$\pi_{S_2}(g^5) = 1627(a_1)^2(39 - 4\delta - 2\delta^2 - 3\theta)^2/(B_6)^2$

$\pi_{M_1}(g^5) = (a_1)^2(78 + 5\delta - \delta^2)^2(217 - 16\delta - 8\delta^2)/(B_6)^2$

$\pi_{M_2}(g^5) = 2(a_1)^2(434 + 8\delta - \delta^2)(39 - 4\delta - 2\delta^2 - 3\theta)^2/(B_6)^2$

where $B_6 = 2\delta^3 + \delta^2(235 - 4\theta - 3\theta^2) + \delta(318 + 23\theta + 15\theta^2) + 6(39\theta^2 + 106\theta - 897)$.

Non-exclusive relations:

$\pi_{S_1}(g^5) = 8(a_1)^2(42 + 11\delta + 2\delta^2)^2(23 - 4\theta - \theta^2)/(B_7)^2$

$\pi_{S_2}(g^5) = 184(a_1)^2[11\delta + 2\delta^2 + 6(\theta - 7)]^2/(B_7)^2$

$\pi_{M_i}(g^5) = 2(a_1)^2(527 - 44\delta - 4\delta^2)(14 - \theta)^2/(B_7)^2$

where $B_7 = 1946 - 349\theta - 84\theta^2 - 4\delta^2(7 + 2\theta + \theta^2) - 2\delta(91 + 21\theta + 11\theta^2)$.

The pattern 6 network (g^6)

Exclusive relations:

$\pi_{S_1}(g^6) = 8(a_1)^2(39 + 5\theta - \theta^2)^2(209 - 32\theta - 16\theta^2)/(B_8)^2$

$\pi_{S_2}(g^6) = 2(a_1)^2(209 + 8\theta - \theta^2)(78 - 3\delta - 16\theta - 8\theta^2)^2/(B_8)^2$

$\pi_{M_1}(g^6) = 2(a_1)^2(434 - 24\delta - 9\delta^2)(39 + 5\theta - \theta^2)^2/(B_8)^2$

$\pi_{M_2}(g^6) = 217(a_1)^2(78 - 3\delta - 16\theta - 8\theta^2)^2/(B_8)^2$

where $B_8 = 5382 - 636\theta - 478\theta^2 - 8\theta^3 - 3\delta^2(39 + 5\theta - \theta^2) - \delta(318 + 23\theta - 4\theta^2)$.

Non-exclusive relations:

$\pi_{S_i}(g^6) = 18(a_1)^2(25 - 2\delta)^2(92 - 8\theta - \theta^2)/(B_9)^2$

$\pi_{M_1}(g^6) = 2(a_1)^2(527 - 88\delta - 16\delta^2)(25 + 12\theta + 3\theta^2)^2/(B_9)^2$

$\pi_{M_2}(g^6) = 1054(a_1)^2(25 - 4\delta - 12\theta - 3\theta^2)^2/(B_9)^2$

where $B_9 = 4\delta^2(25 + 12\theta + 3\theta^2) + 25(15\theta + 3\theta^2 - 139) + 3\delta(201 + 42\theta + 11\theta^2)$.

The pattern 7 network (g^7)

Exclusive relations:

$\pi_{S_1}(g^7) = 2(a_1)^2(78 - 3\delta - 16\theta - 8\theta^2)^2(209 - 24\theta - 9\theta^2)/(B_{10})^2$

$\pi_{S_2}(g^7) = 8(a_1)^2(39 - 4\delta - 2\delta^2 - 3\theta)^2(209 - 32\theta - 16\theta^2)/(B_{10})^2$

$\pi_{M_1}(g^7) = (a_1)^2(217 - 16\delta - 8\delta^2)(78 - 3\delta - 16\theta - 8\theta^2)^2/(B_{10})^2$

$\pi_{M_2}(g^7) = 2(a_1)^2(434 - 24\delta - 9\delta^2)(39 - 4\delta - 2\delta^2 - 3\theta)^2/(B_{10})^2$

where $B_{10} = 26\delta^3 + 6\delta^4 + \delta^2(16\theta^2 + 41\theta - 305) + \delta(41\theta^2 + 97\theta - 750) + 2(2691 - 750\theta - 277\theta^2 + 52\theta^3 + 12\theta^4)$.

Non-exclusive relations:

$\pi_{S_1}(g^7) = 8(a_1)^2(23 - 4\theta - \theta^2)(F_2)^2/(B_{11})^2$

$\pi_{S_2}(g^7) = 2(a_1)^2(92 - 8\theta - \theta^2)[6\delta^2 - 8\delta^3 - 300(7 - \theta) + \delta(527 - 36\theta)]^2/(B_{11})^2$

$\pi_{M_1}(g^7) = 2(a_1)^2(527 - 44\delta - 4\delta^2)[700 - 243\theta - 18\theta^2 + 6\theta^3 - 4\delta(28 - 3\theta)]^2/(B_{11})^2$

$\pi_{M_2}(g^7) = 2(a_1)^2\left(527 - 88\delta - 16\delta^2\right)(F_3)^2/(B_{11})^2$

where $F_2 = 8\delta^3 + 150(\theta - 14) + 6\delta^2(\theta^2 + 4\theta - 1) + \delta(33\theta^2 + 114\theta - 23)$; $F_3 = -700 - 93\theta - 18\theta^2 + 6\theta^3 + 2\delta^2\theta(4 + \theta) + \delta\left(56 + 38\theta + 11\theta^2\right)$; $B_{11} = 8\delta^4\theta(4 + \theta) + \delta^3(336 + 264\theta + 82\theta^2) + 25(3892 - 10047\theta - 150\theta^2 + 18\theta^3) - \delta^2(2044 + 284\theta - 61\theta^2 - 96\theta^3 - 12\theta^4) - 2\delta(12663 + 117\theta + 92\theta^2 - 207\theta^3 - 33\theta^4)$.

The pattern 8 network (g^8)

Exclusive relations:

$\pi_{S_1}(g^8) = 8(a_1)^2(209 - 24\theta - 9\theta^2)[5\delta - \delta^2 + 2(39 + 5\theta - \theta^2)]^2/(B_{12})^2$

$\pi_{S_2}(g^8) = 72(a_1)^2(26 - \delta - 2\theta)^2(209 + 8\theta - \theta^2)/(B_{12})^2$

$\pi_{M_1}(g^8) = 2(a_1)^2(434 - 24\delta - 9\delta^2)[5\delta - \delta^2 + 2(39 + 5\theta - \theta^2)]^2/(B_{12})^2$

$\pi_{M_2}(g^8) = 18(a_1)^2(434 + 8\delta - \delta^2)(26 - \delta - 2\theta)^2/(B_{12})^2$

where $B_{12} = 11\delta^3 - 3\delta^4 + 2\delta(204 + 46\theta + 11\theta^2) + \delta^2(287 + 22\theta - 12\theta^2) - 4(2691 - 204\theta - 155\theta^2 - 11\theta^3 + 3\theta^4)$.

Non-exclusive relations:

g^8 and g^7 imply the same equilibrium outcomes.

Appendix A2. Proofs

Proof of Proposition 1

Exclusive relations:

We show first that g^c is pairwise stable for the set of all (δ, θ) values in the area between the curves C_1 and C_4. From g^c firms can deviate either to g^7 or g^8. In the former case, we have that $\pi_{M_1}(g^c) > \pi_{M_1}(g^7)$ and $\pi_{S_2}(g^c) > \pi_{S_2}(g^7)$ for all (δ, θ) values in the area between the curves C_1 and C_4, whereas in the latter $\pi_{M_2}(g^c) > \pi_{M_2}(g^8)$ and $\pi_{S_2}(g^c) > \pi_{S_2}(g^8)$ for all δ and $\theta \in (0,1]$.[19] This implies that g^c is pairwise stable in the area between the curves C_1 and C_4. It also shows that g^7 is not pairwise stable in that area, while g^8 is not pairwise stable for all δ and $\theta \in (0,1]$.

We now demonstrate that g^3 is pairwise stable for the set of all (δ, θ) values in the area below the curve C_2 or above the curve C_3. From g^3, the possible deviation of firms is either g^7 or g^1. We have that $\pi_{M_2}(g^3) > \pi_{M_2}(g^7)$ and $\pi_{S_1}(g^3) > \pi_{S_1}(g^7)$ for the set of all (δ, θ) values in the area below the curve C_2 or above the curve C_3; while $\pi_{M_2}(g^3) > \pi_{M_2}(g^1)$ and $\pi_{S_2}(g^3) > \pi_{S_2}(g^1)$ for all δ and $\theta \in (0,1]$. This implies that g^3 is pairwise stable in the area below the curve C_2 or above the curve C_3. It also shows that g^7 is not pairwise stable in that area, and g^1 is never pairwise stable.

Because g^c is pairwise stable for the set of all (δ, θ) values in the area between the curves C_1 and C_4 while g^3 is pairwise stable for the set of all (δ, θ) values in the area below the curve C_2 or above the curve C_3, it follows that g^c and g^3 are both pairwise stable for the set of all (δ, θ) values in the area between the curves C_1 and C_2, as well as in the area between the curves C_3 and C_4.

Next we show that no other network is pairwise stable. g^e is not pairwise stable because $\pi_{M_1}(g^e) < \pi_{M_1}(g^1)$ and $\pi_{S_1}(g^e) < \pi_{S_1}(g^1)$. In g^2, firms have an incentive to deviate to g^5 because $\pi_{M_1}(g^5) > \pi_{M_1}(g^2)$ and $\pi_{S_1}(g^5) > \pi_{S_1}(g^2)$. Likewise, in g^5, firms have an incentive to deviate to g^7 because $\pi_{M_2}(g^7) > \pi_{M_2}(g^5)$ and $\pi_{S_2}(g^7) > \pi_{S_2}(g^5)$. Moreover, in g^6, S_2 and M_2 have an incentive to establish a link between them if $\pi_{M_2}(g^6{+}22) > \pi_{M_2}(g^6)$ and $\pi_{S_2}(g^6{+}22) > \pi_{S_2}(g^6)$, where $g^6{+}22$ denotes the network obtained when the link between S_2 and M_2 is added to the network g^6. Due to symmetry, $\pi_{M_2}(g^6{+}22) = \pi_{M_1}(g^7)$ and $\pi_{S_2}(g^6{+}22) = \pi_{S_1}(g^7)$. Because $\pi_{M_1}(g^7) > \pi_{M_2}(g^6)$ and $\pi_{S_1}(g^7) > \pi_{S_2}(g^6)$, g^6 is

[19] Comparisons are performed by means of plots using the software Mathematica 7. Though not presented here, these plots are available on request.

not pairwise stable. Finally, in g^4, firms have an incentive to deviate to g^8 because $\pi_{M_1}(g^4) < \pi_{M_1}(g^8)$ and $\pi_{S_1}(g^4) < \pi_{S_1}(g^8)$. Q.E.D.

Non-exclusive relations:

First, we show that g^c is pairwise stable for all δ and $\theta \in (0,1]$. From g^c firms can deviate either to g^7 or g^8. In the former case, we have that $\pi_{M_1}(g^c) > \pi_{M_1}(g^7)$ and $\pi_{S_2}(g^c) > \pi_{S_2}(g^7)$ for all δ and $\theta \in (0,1]$, whereas in the latter $\pi_{M_2}(g^c) > \pi_{M_2}(g^8)$ and $\pi_{S_2}(g^c) > \pi_{S_2}(g^8)$ for all δ and $\theta \in (0,1]$. This implies that g^c is pairwise stable for all δ and $\theta \in (0,1]$. It also shows that g^7 and g^8 are not pairwise stable.

We now demonstrate that no other network is pairwise stable. From g^3, the possible deviation of firms is either g^7 or g^1. Because $\pi_{M_2}(g^3) < \pi_{M_2}(g^7)$, $\pi_{S_1}(g^3) < \pi_{S_1}(g^7)$, $\pi_{M_2}(g^3) > \pi_{M_2}(g^1)$ and $\pi_{S_2}(g^3) > \pi_{S_2}(g^1)$ for all δ and $\theta \in (0,1]$, it follows that neither g^3 nor g^1 is pairwise stable. g^e is not pairwise stable because $\pi_{M_1}(g^e) < \pi_{M_1}(g^1)$ and $\pi_{S_1}(g^e) < \pi_{S_1}(g^1)$. In g^2, firms have an incentive to deviate to g^5 because $\pi_{M_1}(g^5) > \pi_{M_1}(g^2)$ and $\pi_{S_1}(g^5) > \pi_{S_1}(g^2)$. Similarly, in g^5, firms have an incentive to deviate to g^7 given that $\pi_{M_2}(g^7) > \pi_{M_2}(g^5)$ and $\pi_{S_2}(g^7) > \pi_{S_2}(g^5)$. Additionally, in g^6, S_2 and M_2 have an incentive to establish a link between them if $\pi_{M_2}(g^6{+}22) > \pi_{M_2}(g^6)$ and $\pi_{S_2}(g^6{+}22) > \pi_{S_2}(g^6)$, where $g^6{+}22$ denotes the network obtained when the link between S_2 and M_2 is added to the network g^6. Due to symmetry, $\pi_{M_2}(g^6{+}22) = \pi_{M_1}(g^7)$ and $\pi_{S_2}(g^6{+}22) = \pi_{S_1}(g^7)$. Because $\pi_{M_1}(g^7) > \pi_{M_2}(g^6)$ and $\pi_{S_1}(g^7) > \pi_{S_2}(g^6)$, g^6 is not pairwise stable. Finally, in g^4, firms have an incentive to deviate to g^8 because $\pi_{M_1}(g^4) < \pi_{M_1}(g^8)$ and $\pi_{S_1}(g^4) < \pi_{S_1}(g^8)$. Q.E.D.

Proof of Proposition 2

The result follows directly from the comparisons: $W(g^e) < W(g^2) < W(g^4) < W(g^8) < W(g^c)$; $W(g^2) < W(g^i) < W(g^j) < W(g^c)$; $W(g^e) < W(g^1) < W(g^3) < W(g^7) < W(g^c)$; and $W(g^1) < W(g^i) < W(g^j) < W(g^c)$ for all θ and $\delta \in (0,1]$, where $i \in \{5,6\}$ and $j \in \{7,8\}$. Q.E.D.